

INTEGRAL ESTIMATION IN QUANTUM PHYSICS

by
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The University of Utah Graduate School

STATEMENT OF DISSERTATION APPROVAL

The dissertation of Jane Doe
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the Department/College/School of Mathematics
and by Alice B. Toklas , Dean of The Graduate School.

For my parents, Alice and Bob.

CONTENTS

ABSTRACT	iii
LIST OF FIGURES	vi
LIST OF TABLES	vii
NOTATION AND SYMBOLS	viii
CHAPTERS	
1. THE FIRST	1
1.1 The first section	1
1.1.1 The first subsection	2
1.1.2 The second subsection	2
1.1.3 The third subsection	2
1.1.3.1 The first subsubsection	2
1.1.3.2 The second subsubsection	2
1.1.3.2.1 The first numbered paragraph	2
1.1.3.2.2 The second numbered paragraph	3
1.2 The second section	3
1.3 The third section	5
1.4 Free software packages	6
1.5 Resizing figures	9
1.6 Summary and conclusions	13
2. THE SECOND	15
3. THE THIRD	16
4. THE FOURTH	17
4.1 More on the topic	17
4.2 Even more on the topic	17
4.3 Summary and conclusions	18
APPENDICES	
A. THE FIRST	19
B. THE SECOND	20
C. THE THIRD	21
REFERENCES	24

LIST OF FIGURES

1.1	The first figure.	2
1.2	The second figure.	3
1.3	The third figure.	8
1.4	The fourth figure (at 50% scale).	10
1.5	The fifth figure (at 75% scale).	10
1.6	The sixth figure (at native size).	10
1.7	The seventh figure (at 125% scale).	10
1.8	The eighth figure (at 175% scale).	10
1.9	The ninth figure (at 50% scale)	12
1.10	The tenth figure (at 75% scale)	12
1.11	Using \LaTeX picture mode	13

LIST OF TABLES

1.1	Lowercase Greek letters.	4
1.2	Uppercase Greek letters.	6

NOTATION AND SYMBOLS

α	fine-structure (dimensionless) constant, approximately $1/137$
α	radiation of doubly-ionized helium ions, He ⁺⁺
β	radiation of electrons
γ	radiation of very high frequency, beyond that of X rays
γ	Euler's constant, approximately 0.577 215 ...
δ	stepsize in numerical integration
$\delta(x)$	Dirac's famous function
ϵ	a tiny number, usually in the context of a limit to zero
$\zeta(x)$	the famous Riemann zeta function
...	...
$\psi(x)$	logarithmic derivative of the gamma function
ω	frequency

CHAPTER 1

THE FIRST

This is a chapter. Remember that there should *always* be at least of few lines of prose after each sectional heading: failure to do so is a disservice to your readers, and also produces incorrect vertical spacing.

1.1 The first section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.1** on the following page, we have a picture, and the L^AT_EX markup to include it looks like this:

```
\begin{figure}[t]
  \centerline{\includegraphics{fig1}}
  \caption{The first figure.}%
  \figlabel{fig1}
\end{figure}
```

We intentionally omitted an extension on the filename, so that this document can be processed with `latex` to get an output `.dvi` file, or with `pdflatex` to get an output `.pdf` file. The first case uses the file `fig1.eps`, and the second uses `fig1.pdf`. The `distill` or `ps2pdf` commands can be used to convert from *Encapulated PostScriptfiles* to *Portable Document Formatfiles*.

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This is Figure 1

Figure 1.1. The first figure.

1.1.1 The first subsection

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1.1.2 The second subsection

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1.1.3 The third subsection

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1.1.3.1 The first subsubsection

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1.1.3.2 The second subsubsection

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1.1.3.2.1 The first numbered paragraph Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah.

1.1.3.2 The second numbered paragraph Blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah.

1.2 The second section

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In **Figure 1.2**, we have another picture.

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Blah blah blah.



Figure 1.2. The second figure.

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In **Table 1.1**, we show the 24-character lowercase Greek alphabet.

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 blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Table 1.1. Lowercase Greek letters.

α	alpha
β	beta
γ	gamma
δ	delta
ϵ, ε	epsilon
ζ	zeta
η	eta
θ, ϑ	theta
ι	iota
κ	kappa
λ	lambda
μ	mu
ν	nu
ξ	xi
\omicron	omicron
π	pi
ρ	rho
σ, ς	sigma
τ	tau
υ	upsilon
ϕ, φ	phi
χ	chi
ψ	psi
ω	omega

Table 1.2. Uppercase Greek letters. Notice that several have the same letter shapes as Latin letters, and for those, T_EX does not define macro names. For convenience, we supply our own definitions of these macros: `\Alpha`, `\Beta`, `\Epsilon`, `\Zeta`, `\Eta`, `\Iota`, `\Kappa`, `\Mu`, `\Nu`, `\Omicron`, `\Rho`, `\Tau`, and `\Chi`.

A	Alpha
B	Beta
Γ	Gamma
Δ	Delta
E	Epsilon
Z	Zeta
H	Eta
Θ	Theta
I	Iota
K	Kappa
Λ	Lambda
M	Mu
N	Nu
Ξ	Xi
O	Omicron
Π	Pi
P	Rho
Σ	Sigma
T	Tau
Y	Upsilon
Φ	Phi
X	Chi
Ψ	Psi
Ω	Omega

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.4 Free software packages

The Free Software Foundation offers almost 300 software packages, most easily portable to many different operating systems and CPU platforms. They include at least these:

`a2ps`, `acct`, `acm`, `adns`, `alive`, `anubis`, `apl`, `archimedes`, `aris`, `aspell`, `auctex`, `autoconf-archive`, `autoconf`, `autogen`, `automake`, `avl`, `ballandpaddle`, `barcode`, `bash`, `bayonne`, `bc`, `binutils`, `bison`, `bool`, `bpel2owfn`, `c-graph`, `ccaudio`, `ccd2cue`, `ccrtp`,

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1.5 Resizing figures

In **Figure 1.4** through **Figure 1.8** on the following page, we show how graphics files can be rescaled to convenient sizes, with input like this:

```
\begin{figure}[p]
  \centerline{\includegraphics[scale = 0.5]{fig1}}
  \caption{The fourth figure (at 50\% scale).}%
  \figlabel{fig4}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 0.75]{fig1}}
  \caption{The fifth figure (at 75\% scale).}%
  \figlabel{fig5}
\end{figure}
```



This is Figure 1

Figure 1.4. The fourth figure (at 50% scale).



This is Figure 1

Figure 1.5. The fifth figure (at 75% scale).



This is Figure 1

Figure 1.6. The sixth figure (at native size).



This is Figure 1

Figure 1.7. The seventh figure (at 125% scale).



This is Figure 1

Figure 1.8. The eighth figure (at 175% scale).



This is Figure 1

Figure 1.9. The ninth figure (at 50% scale), boxed with the tenth figure.



This is Figure 1

Figure 1.10. The tenth figure (at 75% scale), boxed with the ninth figure.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah
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As a final example in this chapter, **Figure 1.11** on the following page shows how you can use \LaTeX picture mode for annotating and positioning graphics images prepared outside \LaTeX . The input that produced that figure looks like this:

```
\begin{figure}[t]
  %% The original image is 216bp wide by 72bp high, but we
  %% rescale it to 150 picture units divided by \unitlength:
  %% 150 / 0.75 = 112.5 mm
  \newcommand {\myfig} {\includegraphics[width = 112.5mm]{fig1}}

  \begin{center}
    %% The \unitlength is chosen to make the complete picture fit
    %% within the page margins

    \setlength{\unitlength}{0.75mm}

    %%%      insert (width,height)(lower-left-x,lower-left-y)
    \begin{picture}(170,70)(10,10)
      %% Place the included image FIRST!
      \put(10,10) {\myfig}

      %% Everything that follows OVERLAYS the original image!

      \graphpaper[10](0,0)(170,70)

      %% Mark the image center and corners by centered bullets
      \newcommand {\thedot} {\makebox (0,0) {$\bullet$}}
      \put( 85, 35) {\thedot}
      \put( 10, 10) {\thedot}
```


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blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

APPENDIX C

THE THIRD

This is an appendix.

There are several books [12, 19–21, 23–25, 27–30] listed in our bibliography.

We also reference several journal articles [1, 2, 4, 8–10, 13–18, 22, 31, 32] and three famous doctoral theses of later winners [3, 6, 7] of the Nobel Prize in Physics (1922, 1933, and 1921):

Notice that, even though those citations appeared in \LaTeX `\cite{...}` commands with their BibTeX citation labels in reverse alphabetical order, thanks to the `citesort` package, their reference-list numbers have been sorted in numerically ascending order, and then range-reduced.

Mention should also be made of a famous Dutch computer scientist's first publication [5].

Font metrics are an important, albeit low-level, aspect of typesetting. See the *Adobe Systems* manual about that company's procedures [26].

The bibliography at the end of this thesis contains several examples of documents with non-English titles, and their BibTeX entries provide title translations following the practice recommended by the American Mathematical Society and SIAM. Here is a sample entry that shows how to do so:

```
@PhdThesis{Einstein:1905:NBM,  
  author =      "Albert Einstein",  
  title =       "{Eine Neue Bestimmung der Molek{\u}ldimensionen}.  
                ({{German}}) [{{A}} new determination of molecular  
                dimensions]",  
  type =        "Inaugural dissertation",  
  school =      "Bern Wyss.",  
  address =     "Bern, Switzerland",  
  year =        "1905",  
  bibdate =     "Fri Dec 17 10:46:57 2004",
```

```

bibsource = "http://www.math.utah.edu/pub/tex/bib/einstein.bib",
note = "Published in \cite{Einstein:1906:NBM}.",
acknowledgement = ack-nhfb,
language = "German",
advisor = "Alfred Kleiner (24 April 1849--3 July 1916)",
URL = "http://en.wikipedia.org/wiki/Alfred_Kleiner",
remark = "Received August 19, 1905 and published February 8,
1906.",
Schilpp-number = "6",
}

```

The note field in that entry refers to another bibliography entry that need not have been directly cited in the document text. Such cross-references are common in `BIBTEX` files, especially for journal articles where there may be later comments and corrigenda that should be mentioned. Embedded `\cite{}` commands ensure that those possibly-important other entries are always included in the reference list when the entry is cited. The last bibliography entry [32] in this thesis has a long note field that tells more about what some may view as the most important paper in mathematics in the last century.

When entries cite other entries that cite other entries that cite other entries that ..., multiple passes of `LATEX` and `BIBTEX` are needed to ensure consistency. That is another reason why document compilation should be guided by a `Makefile` or a batch script, rather than expecting the user to remember just how many passes are needed.

`BIBTEX` entries are *extensible*, in that arbitrary key/value pairs may be present that are not necessarily recognized by any bibliography style files. The `advisor`, `acknowledgement`, `bibdate`, `bibsource`, `language`, `remark`, and `Schilpp-number` fields are examples, and may be used by other software that processes `BIBTEX` entries, or by humans who read the entries. `DOI` and `URL` fields are currently recognized by only a few styles, but that situation will likely change as publishers demand that such important information be included in reference lists.

In `BIBTEX` `title` fields, braces protect words, such as proper nouns and acronyms, that cannot be downcased if the selected bibliography style would otherwise do so. In German, all nouns are capitalized, and the simple way to ensure their protection is to brace the entire German text in the title, as we did in the entry above.

The world's first significant computer program may have been that written in 1842

by Lady Augusta Ada Lovelace (1815–1852) for the computation of Bernoulli numbers [16, 18]. She was the assistant to Charles Babbage (1791–1871), and they are the world’s first computer programmers. The programming language *Ada* is named after her, and is defined in the ANSI/MIL-STD-1815A Standard; its number commemorates the year of her birth.

We do not discuss mathematical *transforms* in this dissertation, but you can find that phrase in the index (except that this sample thesis doesn’t have one!)

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- [10] A. EINSTEIN, *Berichtigung zu meiner Arbeit: Eine neue Bestimmung der Moleküldimensionen. (German) [Corrections to my work: a new determination of molecular dimensions]*, Annalen der Physik (1900) (series 4), 339 (1911), pp. 591–592. See [8].
- [11] ———, *Eine Beziehung zwischen dem elastischen Verhalten und der spezifischen Wärme bei festen Körpern mit einatomigem Molekül. (German) [A relationship between the elastic behavior and the specific heat of solid bodies with monatomic molecules]*, Annalen der Physik (1900) (series 4), 339 (1911), pp. 170–174, 590. See remarks [9, 10].

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- [32] A. WILES, *Modular elliptic curves and Fermat's Last Theorem*, *Annals of Mathematics*, 142 (1995), pp. 443–551. This paper contains the bulk of the author's proof of the Taniyama–Shimura conjecture and Fermat's Last Theorem, carried out at Princeton University. The companion paper [31] contains the solution to the flaw discovered in the proof that Wiles announced on June 23, 1993, in Cambridge, England. See also [30]. In March 2014, now Royal Society Research Professor Sir Andrew John Wiles of Oxford University was awarded the prestigious Abel Prize in Mathematics for this proof — an award that also carries a cash prize of six million Norwegian crowns, or about US\$722,000.