# QUADRATIC AND CUBIC FUNCTIONS DEFINED ON THE POSITIVE INTEGERS

by Stuart Dent

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The University of Arizona

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SIGNED:

# DEDICATION

For my mom.

#### Acknowledgments

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#### Abstract

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Stuart Dent, Ph.D. The University of Arizona, 2010

Director: Anna Mehmbuhr

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#### Chapter 1

#### CHAPTER TITLE

#### **1.1** Disclaimers and usage instructions

This is a version of my PhD dissertation which I defended and got through Graduate College format review in spring 2010. I hope it works for you. Please keep in mind, however, that style policies may change. Of course, please consult the style information you received from the Graduate College.

I should acknowledge that I didn't write this from scratch. I got this template from I forget where — somewhere in LPL, I think — and modified to my taste, and to match current formatting requirements. So, this is a current snapshot of collectively developed dissertation style for the University of Arizona.

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- Search around in stu-dent-dis.tex, ua-thesis.cls, disack.tex, etc. for the names "Stuart Dent" and "Anna Mehmbuhr", etc. Change these to your name and the names of your committee members. Also change the title of your dissertation.
- In those same files, look for anything else which needs changing.
- In the first line of stu-dent-dis.tex, you can set the style to draft or final.
- The shell scripts ./dbuild and ./pbuild, in the same directory as this file, can be used to create DVI and PDF files, respectively.
- Any LAT<sub>F</sub>X problem you have is most likely resolvable by a Google search.
- Good luck, and have fun!
- John "Stu Dent" Kerl, April 21, 2010.

#### **1.2** Section title

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

Table 1.1 will be discussed in section 1.3.

x	f(x)
0	0
1	1
2	4
3	9
4	16
5	25

TABLE 1.1. Here is the long caption for the body of the document. Note that it's OK for this to have many lines. But the short caption for the list of tables should be short. Hence the difference between square brackets and curly braces in the caption command in the LATEX code. And, you guessed it — this last sentence is here just to make this caption even longer.

#### **1.3** Section title

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox

x	g(x)
0	0
1	1
2	8
3	27
4	64
5	125

TABLE 1.2. Here is another caption.

jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

Compare table 1.2 with table 1.1 which was on page 11. We conjecture, but are unable to prove, a relationship between these tabular data and the function of equation (1.3.1):

$$y = x^4 - x^2 + 1. (1.3.1)$$

### Chapter 2 CHAPTER TITLE

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

Please see figure 2.1.

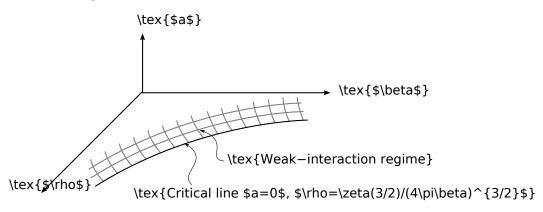


FIGURE 2.1. Here is an EPS file which I put in here and which surely must be important.

#### 2.1 Section title

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown *fox* jumped over the lazy *dogs*. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

#### 2.2 Section title

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

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