1

THE BINOMIAL MODEL

In this chapter we will study, in some detail, the simplest possible nontrivial model of a financial market—the binomial model. This is a discrete time model, but despite the fact that the main purpose of the book concerns continuous time models, the binomial model is well worth studying. The model is very easy to understand, almost all important concepts which we will study later on already appear in the binomial case, the mathematics required to analyze it is at high school level, and last but not least the binomial model is often used in practice.

1.1 The One Period Model

We start with the one period version of the model. In the next section we will (easily) extend the model to an arbitrary number of periods.

1.1.1 Model Description

Running time is denoted by the letter t, and by definition we have two points in time, $t = 0$ ("today") and $t = 1$ ("tomorrow"). In the model we have two assets: a **bond** and a **stock**. At time t the price of a bond is denoted by B_t, and the price of one share of the stock is denoted by S_t. Thus we have two price processes B and S.

1. The bond price process is deterministic and given by
   a. This is a discrete time model, but despite the fact that the main purpose of the book concerns continuous time models, the binomial model is well worth studying.
   b. The model is very easy to understand, almost all important concepts which we will study later on already appear in the binomial case, the mathematics required to analyze it is at high school level, and last but not least the binomial model is often used in practice.
2. The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The stock price process is a stochastic process, and its dynamical behavior is described as follows:

$$S_0 = s, \tag{1.1}$$

$$S_1 = \begin{cases} s \cdot u, & \text{with probability } p_u. \\ s \cdot d, & \text{with probability } p_d. \end{cases} \tag{1.2}$$

It is often convenient to write this as

$$\begin{cases} S_0 = s, \\ S_1 = s \cdot Z, \end{cases}$$

where Z is a stochastic variable defined as[1]

$$Z = \begin{cases} u, & \text{with probability } p_u. \\ d, & \text{with probability } p_d. \end{cases}$$

- We[2] assume that today's stock price s is known, as are the positive constants u, d, p_u and p_d. We assume that $d < u$, and we have of course $p_u + p_d = 1$. We can illustrate the price dynamics using the tree structure in Fig. 1.1.
- We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$.

For Margin
Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory

[1] We assume that today's stock price s is known, constants u, d, p_u and p_d. We assume that $d < u$, and we have of course $p_u + p_d = 1$.

[2] We assume that $d < u$, and we have of

$$a + b = c$$

course $p_u + p_d = 1$.

Definition 1.1 *An* **arbitrage** *portfolio is a portfolio h with the properties*

$$V_0^h = 0, \tag{1.3}$$

$$V_1^h > 0, \quad \textit{with probability } 1. \tag{1.4}$$

An arbitrage portfolio is thus basically a deterministic money making machine, and we interpret the existence of an arbitrage portfolio as equivalent to a serious case of mispricing on the market. It is now natural to investigate when a given market model is arbitrage free, i.e. when there are no arbitrage portfolios.

Proposition 1.2 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.5}$$

The condition (1.25) has an easy economic interpretation. It simply says that the return on the stock is not allowed to dominate the return on the bond and vice versa.

Now assume that (1.25) is satisfied. To show that this implies absence of arbitrage let us consider an arbitrary portfolio such that $V_0^h = 0$. We thus have $x + ys = 0$, i.e. $x = -ys$. Using this relation we can write the value of the portfolio at $t = 1$ as

$$V_1^h = \begin{cases} ys(u - (1+R)), & \text{if } Z = u. \\ ys(d - (1+R)), & \text{if } Z = d. \end{cases}$$

Assume now that $y > 0$. Then h is an arbitrage strategy if and only if we have the inequalities

$$u > 1 + R, \tag{1.6}$$

$$d > 1 + R, \tag{1.7}$$

but this is impossible because of the condition (1.25). The case $y < 0$ is treated similarly.

At first glance this result is perhaps only moderately exciting, but we may write it in a more suggestive form. To say that (1.25) holds is equivalent to saying that $1 + R$ is a convex combination of u and d, i.e.

$$1 + R = q_u \cdot u + q_d \cdot d,$$

where $q_u, q_d \geq 0$ and $q_u + q_d = 1$. In particular we see that the weights q_u and q_d can be interpreted as probabilities for a new probability measure Q

with the property $Q(Z = u) = q_u$, $Q(Z = d) = q_d$. Denoting expectation w.r.t. this measure by E^Q we now have the following easy calculation

$$\frac{1}{1+R}QS_1 = 11 + R(q_u su + q_d sd) = \frac{1}{1+R} \cdot s(1+R) = s.$$

We thus have the relation

$$s = \frac{1}{1+R}S_1,$$

which to an economist is a well-known relation. It is in fact a **risk neutral** valuation formula, in the sense that it gives today's stock price as the discounted expected value of tomorrow's stock price. Of course we do not assume that the agents in our market are risk neutral—what we have shown is only that if we use the Q-probabilities instead of the objective probabilities then we have in fact a risk neutral valuation of the stock (given absence of arbitrage). A probability measure with this property is called a **risk neutral measure**, or alternatively a **risk adjusted measure** or a **martingale measure**. Martingale measures will play a dominant role in the sequel so we give a formal definition.

Definition 1.3 *A probability measure Q is called a* **martingale measure** *if the following condition holds:*

$$S_0 = \frac{1}{1+R}S_1.$$

We may now state the condition of no arbitrage in the following way.

For the binomial model it is easy to calculate the martingale probabilities. The proof is left to the reader.

1.1.2 Contingent Claims

Let us now assume that the market in the preceding section is arbitrage free. We go on to study pricing problems for contingent claims.

Definition 1.4 *A* **contingent claim** *(financial derivative) is any stochastic variable X of the form $X = \Phi(Z)$, where Z is the stochastic variable driving the stock price process above.*

We interpret a given claim X as a contract which pays X SEK to the holder of the contract at time $t = 1$. See Fig. **??**, where the value of the claim at each node is given within the corresponding box. The function Φ is called the **contract function**. A typical example would be a European call option on the stock with strike price K. For this option to be interesting we assume that $sd < K < su$. If $S_1 > K$ then we use the option, pay K to get the stock and then sell the stock on the market for su, thus making a net profit of $su - K$. If $S_1 < K$ then the option is obviously worthless. In this example we thus have

$$X = \begin{cases} su - K, & \text{if } Z = u, \\ 0, & \text{if } Z = d, \end{cases}$$

and the contract function is given by

$$\Phi(u) = su - K, \tag{1.8}$$

$$\Phi(d) = 0. \tag{1.9}$$

Our main problem is now to determine the "fair" price, if such an object exists at all, for a given contingent claim X. If we denote the price of X at time t by X, then it can be seen that at time $t = 1$ the problem is easy to solve. In order to avoid arbitrage we must (why?) have

$$X = X,$$

and the hard part of the problem is to determine X. To attack this problem we make a slight detour.

Since we have assumed absence of arbitrage we know that we cannot make money out of nothing, but it is interesting to study what we **can** achieve on the market.

Definition 1.5 *A given contingent claim X is said to be* **reachable** *if there exists a portfolio h such that*

$$V_1^h = X,$$

with probability 1. *In that case we say that the portfolio h is a* **hedging** *portfolio or a* **replicating** *portfolio. If all claims can be replicated we say that the market is* **complete**.

If a certain claim X is reachable with replicating portfolio h, then, from a financial point of view, there is no difference between holding the claim and holding the portfolio. No matter what happens on the stock market, the value of the claim at time $t = 1$ will be exactly equal to the value of the portfolio at $t = 1$. Thus the price of the claim should equal the market value of the portfolio, and we have the following basic pricing principle.

The word "reasonable" above can be given a more precise meaning as in the following proposition. We leave the proof to the reader.

We see that in a complete market we can in fact price all contingent claims, so it is of great interest to investigate when a given market is complete. For the binomial model we have the following result.

Proof. We fix an arbitrary claim X with contract function Φ, and we want to show that there exists a portfolio $h = (x, y)$. □

1.1.3 Risk Neutral Valuation

Since the binomial model is shown to be complete we can now price any contingent claim.

Proposition 1.6 *If the binomial model is free of arbitrage, then the arbitrage free price of a contingent claim X is given by*

$$X = \frac{1}{1+R} X. \tag{1.10}$$

Here the martingale measure Q is uniquely determined by the relation

$$S_0 = \frac{1}{1+R}, \tag{1.11}$$

and the explicit expression for q_u and q_d are given in Proposition **??**. *Furthermore the claim can be replicated using the portfolio*

$$x = \frac{1}{1+R} \cdot \frac{u\Phi(d) - d\Phi(u)}{u-d},$$

$$y = \frac{1}{s} \cdot \frac{\Phi(u) - \Phi(d)}{u-d}.$$

We see that the formula (1.10) is a "risk neutral" valuation formula, and that the probabilities which are used are just those for which the stock itself admits a risk neutral valuation. The main economic moral can now be summarized.

We end by studying a concrete example.

Example 1.7 We set $s = 100$, $u = 1.2$, $d = 0.8$, $p_u = 0.6$, $p_d = 0.4$ and, for computational simplicity, $R = 0$. By convention, the monetary unit is

Table 1.1 Table caption

Possible cuts	1–100 m
Total plasma mass	$10 - 10 - 2$ gm
Ion concentration	$10 - 10$ m -3
Temperature	1–40 keV
Pressure	0.1–5 atmospheres
Ion thermal velocity	100–1000 km s
Electron thermal velocity	$0.01c$–$0.1c$
Magnetic field	1–10 T
Total plasma current	0.1–7 MA

Table footnote

the US dollar. Thus we have the price dynamics

$$S_0 = 100, \tag{1.12}$$

$$S_1 = \begin{cases} 120, & \text{with probability } 0.6. \\ 80, & \text{with probability } 0.4. \end{cases} \tag{1.13}$$

If we compute the discounted expected value (under the objective probability measure P) of tomorrow's price we get assume a constant deterministic short rate of interest R, which is interpreted as the simple period rate. This means that the bond price dynamics are given by

$$B_{n+1} = (1+R)B_n, \tag{1.14}$$

$$B_0 = 1. \tag{1.15}$$

We now go on to define the concept of a dynamic portfolio strategy.

Definition 1.8 *A* **portfolio strategy** *is a stochastic process such that h_t is a function of $S_0, S_1, \ldots, S_{t-1}$. For a given portfolio strategy h we set $h_0 = h_1$ by convention. The* **value process** *corresponding to the portfolio h is defined by*

$$V_t^h = x_t(1+R) + y_t S_t.$$

The condition above is in fact also sufficient for absence of arbitrage, but this fact is a little harder to show, and we will prove it later. In any case we assume that the condition holds.

Lemma 1.9 *If the model is free of arbitrage then the following conditions necessarily must hold.*

$$d \leq (1+R) \leq u. \tag{1.16}$$

It is possible, and not very hard, to give a formal proof of the proposition, using mathematical induction.

Proposition 1.10 *The multiperiod binomial model is complete, i.e. every claim can be replicated by a self-financing portfolio.*

The formal proof will, however, look rather messy with lots of indices, so instead we prove the proposition for a concrete example, using a binomial tree.

Example 1.11 We set $T = 3$, $S_0 = 80$, $u = 1.5$, $d = 0.5$, $p_u = 0.6$, $p_d = 0.4$ and, for computational simplicity, $R = 0$.

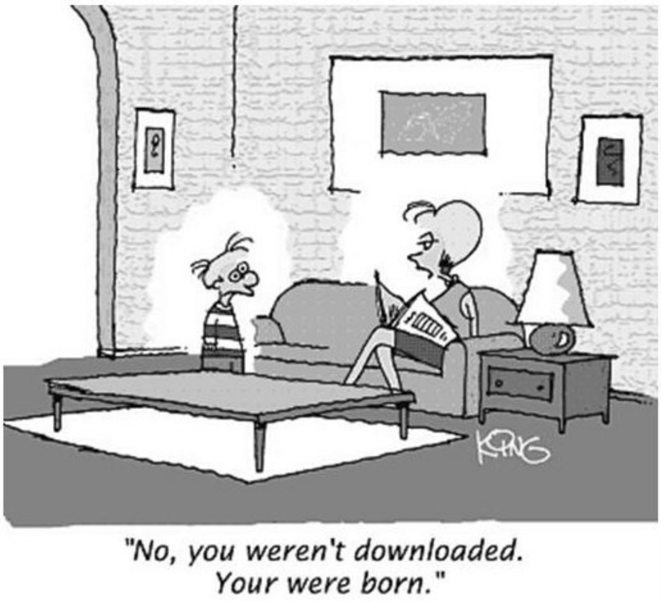

FIGURE 1.1. Price dynamics to this end we define a portfolio as a vector $h = (x, y)$

Unlist

The bond price process is deterministic and given by

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Description

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Verse

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.
The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Quotation

> The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.
> The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Quote

> The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest. The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Theorem 1.12 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.17}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Fundamental Problems 1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.18}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Corollary 1.13 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.19}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Assumption 1.1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.20}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Condition 1.1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.21}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Idea 1.1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.22}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Remark 1.1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.23}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Result 1.1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.24}$$

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Notation convention 1.1.1 *The model above is free of arbitrage if and only if the following conditions hold:*

$$d \leq (1 + R) \leq u. \tag{1.25}$$

1.2 Exercises

The constant R is the spot rate for the period, and we can also interpret the existence of the bond as the existence of a bank with R as its rate of interest.

Boxed Text Crossed the Page

INVESTIGATION

Blind Embedding and Linear Correlation Detection

The embedding algorithm in the system we describe here implements a blind embedder. We denote this algorithm by EBLIND, which refers to this *specific* example of blind embedding rather than the generic concept of blind embedding. In fact, there are many other algorithms for blind embedding.

The detection algorithm uses linear correlation as its detection metric. This is a very common detection metric, which is discussed further in Section 3.5.

To keep things simple, we code only one bit of information. Thus, m is either 1 or 0. We assume that we are working with only grayscale images. Most of the algorithms presented in this book share these simplifications. Methods of encoding more than one bit are discussed in Chapters 4 and 5.

Figure 1.9

Distribution of linear correlations between images and a low-pass filtered random noise pattern.

watermarked image in Figure 3.8 has significantly worse fidelity than that in Figure 3.7, because the human eye is more sensitive to low-frequency patterns than to high-frequency patterns.

If a certain claim X is reachable with replicating portfolio h, then, from a financial point of view, there is no difference between holding the claim and holding the portfolio. No matter what happens on the stock market, the value of the claim at time $t = 1$ will be exactly equal to the value of the portfolio at $t = 1$. Thus the price of the claim should equal the market value of the portfolio, and we have the following basic pricing principle.

If a certain claim X is reachable with replicating portfolio h, then, from a financial point of view, there is no difference between holding the claim and holding the portfolio. No matter what happens on the stock market, the value of the claim at time $t = 1$ will be exactly equal to the value of the portfolio at $t = 1$. Thus the price of the claim should equal the market value of the portfolio, and we have the following basic pricing principle.

If a certain claim X is reachable with replicating portfolio h, then, from a financial point of view, there is no difference between holding the claim and holding the portfolio.

No matter what happens on the stock market, the value of the claim at time $t = 1$ will be exactly equal to the value of the portfolio at $t = 1$. Thus the price of the claim should equal the market value of the portfolio, and we have the following basic pricing principle.

The reader may wish to view (or revisit) the animation "Inductive and Radiative Coupling" on the CD.

Mathematica is very flexible and most calculations can be carried out in more than one way. Depending on how you think, some sequences of calculations may make more sense to you than others, even if they are less efficient than the most efficient way to perform the desired calculations.

Often, the difference in time required for Mathematica to perform equivalent – but different – calculations is quite small. For the beginner, we think it is wisest to work with familiar calculations first and then efficiency.

Example 1.1.13 Calculate (a) $121 + 542$; (b) $3231 - 9876$; (c) $(-23)(76)$; (d) $(22341)(832748)(387281)$; and (e) $\frac{467}{31}$.

Solution These calculations are carried out in the following screen shot. In each case, the input is typed and then evaluated by pressing **Enter**. In the last case, the **Basic Math** template is used to enter the fraction.

Mathematica is very flexible and most calculations can be carried out in more than one way. Depending on how you think, some sequences of calculations may make more sense to you than others, even if they are less efficient than the most efficient way to perform the desired calculations. Often, the difference in time required for Mathematica to perform equivalent – but different – calculations is quite small. For the beginner, we think it is wisest to work with familiar calculations first and then efficiency.

References

[1] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[2] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[3] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[4] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[5] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[6] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[7] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[8] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[9] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[10] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[11] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[12] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[13] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[14] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[15] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[16] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[17] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[18] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[19] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[20] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[21] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

[22] Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

[23] Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

[24] Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

References

Amin, K. & Jarrow, R. (1991). Pricing Foreign Currency Options under Stochastic Interest Rates. *Journal of International Money and Finance* **10**, 310–329.

Anderson, N., Breedon, F., Deacon, M., Derry, A. & Murphy, G. (1996). *Estimating and Interpreting the Yield Curve.* Wiley, Chichester.

Artzner, P. & Delbaen, F. (1989). Term Structure of Interest Rates: The Martingale Approach. *Advances in Applied Mathematics* **10**, 95–129.

Appendix A

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

A.1 Appendix Head1

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

A.2 Appendix Head2

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

Appendix B

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

B.1 Appendix Head1

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

B.2 Appendix Head2

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.

We will study the behavior of various **portfolios** on the (B, S) market, and to this end we define a portfolio as a vector $h = (x, y)$. The interpretation is that x is the number of bonds we hold in our portfolio, whereas y is the number of units of the stock held by us. Note that it is quite acceptable for x and y to be positive as well as negative. If, for example, $x = 3$, this means that we have bought three bonds at time $t = 0$. If on the other hand $y = -2$, this means that we have sold two shares of the stock at time $t = 0$. In financial jargon we have a **long** position in the bond and a **short** position in the stock. It is an important assumption of the model that short positions are allowed.

Consider now a fixed portfolio $h = (x, y)$. This portfolio has a deterministic market value at $t = 0$ and a stochastic value at $t = 1$.

Everyone wants to make a profit by trading on the market, and in this context a so called arbitrage portfolio is a dream come true; this is one of the central concepts of the theory.