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WRITING ASSIGNMENT 1

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Theorem 1. *Let A be any matrix of the form*

$$A = \begin{bmatrix} a & b & \alpha \\ c & d & \beta \end{bmatrix}.$$

Then $ad - bc \neq 0$ if and only if A is row equivalent to

$$I = \begin{bmatrix} 1 & 0 & \gamma \\ 0 & 1 & \delta \end{bmatrix}.$$

Proof.

$$A = \begin{bmatrix} a & b & \alpha \\ c & d & \beta \end{bmatrix}.$$

$R_1 * \frac{1}{a}$ replaces R_1

$$A = \begin{bmatrix} 1 & \frac{b}{a} & \frac{\alpha}{a} \\ c & d & \beta \end{bmatrix}.$$

$R_2 - R_1 * c$ replaces R_2

$$A = \begin{bmatrix} 1 & \frac{b}{a} & \frac{\alpha}{a} \\ 0 & \frac{d-bc}{a} & \beta - \frac{\alpha * c}{a} \end{bmatrix}.$$

$R_2 * a$ replaces R_2

$$A = \begin{bmatrix} 1 & \frac{b}{a} & \frac{\alpha}{a} \\ 0 & ad - bc & \beta * a - \alpha * c \end{bmatrix}.$$

$R_2 * \frac{1}{ad-bc}$ replaces R_2

$$A = \begin{bmatrix} 1 & \frac{b}{a} & \frac{\alpha}{a} \\ 0 & 1 & \frac{\beta * a - \alpha * c}{ad-bc} \end{bmatrix}.$$

$R_1 * a$ replaces R_1

$$A = \begin{bmatrix} a & b & \alpha \\ 0 & 1 & \frac{\beta * a - \alpha * c}{ad-bc} \end{bmatrix}.$$

$R_1 - R_2 * b$ replaces R_1

$$A = \begin{bmatrix} a & 0 & \alpha - \frac{ab*\beta - bc\alpha}{ad-bc} \\ 0 & 1 & \frac{\beta * a - \alpha * c}{ad-bc} \end{bmatrix}.$$

$R_1 * \frac{1}{a}$ replaces R_1

$$A = \begin{bmatrix} 1 & 0 & \frac{\alpha}{a} - \frac{ab*\beta - bc*\alpha}{a(ad-bc)} \\ 0 & 1 & \frac{\beta * a - \alpha * c}{ad-bc} \end{bmatrix}.$$

Assume $\gamma = \frac{\alpha}{a} - \frac{ab*\beta - bc*\alpha}{a(ad-bc)}$
Assume $\delta = \frac{\beta*a-\alpha*c}{ad-bc}$
$ad - bc \neq 1$ □