**Theorem 2.3.** DeMorgan's Laws for sets. Let A and B be sets. Then we have

1. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
2. $\overline{A \cap B} = \overline{A} \cup \overline{B}$

*Proof.* To prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$ , we start by showing that each set is a subset of the other.The definition of a subset states that A is a subset of B if every element $a \in A$ is also an element of B.Thus if two sets have the same elements, since A and B are sets, if $A \subset B$ and $B \subset A$, then A = B. Suppose $x \in \overline{A \cup B}$ , which means $x \notin A \cup B$. Then $x \notin A$ or $x \notin B$. Hence, $x \in \overline{A}$ or $x \in \overline{B}$. This means $x \in \overline{A} \cap \overline{B}$. Thus, $\overline{A \cup B} \subset \overline{A} \cap \overline{B}$. Now suppose, $x \in \overline{A} \cap \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. Hence $x \notin A$ or $x \notin B$, Which means that $x \notin A \cup B$. Therefore, $x \in \overline{A \cup B}$. Thus proving that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

To prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ , we start by showing that each set is a subset of the other. Suppose $x \in \overline{A \cap B}$ , which means $x \notin A \cap B$. Then $x \notin A$ or $x \notin B$. Hence, $x \in \overline{A}$ or $x \in \overline{B}$. This means $x \in \overline{A} \cup \overline{B}$. Thus, $\overline{A \cap B} \subset \overline{A} \cup \overline{B}$. Now suppose, $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$. Hence $x \notin A$ or $x \notin B$, Which means that $x \notin A \cap B$. Therefore, $x \in \overline{A \cap B}$. Thus proving that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

■

**Theorem 4.4.** Let a,b and c $\in \mathbb{Z}$. If $\frac{a}{b}$ and $\frac{b}{c}$ then $\frac{a}{c}$.

*Proof.* Assume $\frac{a}{b}$ and $\frac{b}{c}$. Since $\frac{a}{b}$, there exists $n1 \in \mathbb{Z}$ such that a$n$1=b. Since $\frac{b}{c}$, there exists $n2$ such that b$n$2=c.Since we know the existential statement is true in the universe you can use it to create an instance of an object with the property it describes. So, we let m=n1n2.Then am=an1n2=bn2=c.Since am=c, we have shown $\frac{a}{c}$.

■