

# Is $e + \pi$ irrational?

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## Introduction

In mathematics, a rational number is any number that can be expressed as the quotient or fraction  $p/q$  of two integers, a numerator  $p$  and a non-zero denominator  $q$ . Since  $q$  may be equal to 1, every integer is a rational number. The set of all rational numbers, often referred to as "the rationals", is usually denoted by a boldface  $\mathbb{Q}$  (or blackboard bold  $\mathbb{Q}$ , Unicode  $\mathbb{Q}$ ); it was thus denoted in 1895 by Giuseppe Peano after *quoziente*, Italian for "quotient". The decimal expansion of a rational number always either terminates after a finite number of digits or begins to repeat the same finite sequence of digits over and over. Moreover, any repeating or terminating decimal represents a rational number. These statements hold true not just for base 10, but also for any other integer base (e.g. binary, hexadecimal). A real number that is not rational is called irrational. Irrational numbers include  $\sqrt{2}$ ,  $\ln 2$ ,  $e$ , and  $\pi$ . The decimal expansion of an irrational number continues without repeating. Since the set of rational numbers is countable, and the set of real numbers is uncountable, almost all real numbers are irrational.

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## Definition

### Definition 1

The number  $\pi$  is a mathematical constant, the ratio of a circle's circumference to its

diameter, commonly approximated as 3.14159 and so on so forth. It has been represented by the Greek letter "π" since the mid-18th century, though it is also sometimes spelled out as "pi" (/pa /).

Definition 2

The number e is an important mathematical constant that is the base of the natural logarithm. It is approximately equal to 2.71828 and so on and so forth and is the limit of  $(1 + 1/n)^n$  as n approaches infinity, an expression that arises in the study of compound interest. It can also be calculated as the sum of the infinite series

Body

In this article, we would tackle the rationality of  $e + \pi$ . We know that both e and  $\pi$  are irrational but we cannot surely say that  $e + \pi$  is also irrational since adding them might give a rational number. Now, e and  $\pi$  are rather peculiar numbers. It turns out that, in addition to being irrational numbers, they are also transcendental numbers. Basically, a number is transcendental if there are n polynomials with rational coefficients that have that number as a root.

Now, suppose  $e + \pi$  is rational number then their decimal notation would either be repeating or terminating so it must be repeating,

$$e + \pi = 5.p_1p_2...P_n p_1p_2...P_n p_1p_2...P_n \text{ where } n, p \in Z \text{ and } p_1=8 \text{ and } p_2=5$$

$$e = 2.b_1b_2...b_n e_1e_2...e_n f_1f_2...f_n \text{ where } n, b, e, f \in Z \text{ and } b_1=7 \text{ and } b_2=1$$

$$\pi = 3.a_1a_2...a_n c_1c_2...c_n d_1d_2...d_n \text{ where } n, a, c, d \in Z \text{ and } a_1=1 \text{ and } a_2=4$$

$$p_1p_2...P_n = b_1b_2...b_n + a_1a_2...a_n$$

$$p_1p_2...P_n = e_1e_2...e_n + c_1c_2...c_n$$

$$p_1p_2...P_n = f_1f_2...f_n + d_1d_2...d_n$$

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(then the combination of this infinite number come to a point where in the cycle repeats. but e and  $\pi$  is a transcendental number so which means it contradict our hypothesis that  $e + \pi$  is rational number then by contradiction,  $e + \pi$  is irrational.)

*<https://en.wikipedia.org/wiki/Pi>*

*[https://en.wikipedia.org/wiki/Rational\\_number](https://en.wikipedia.org/wiki/Rational_number)*