Introduction to Quantum Mechanics:
An Overview

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Abstract

This abstract section will serve as an introduction, and an explanation of variables and diagrams used in this paper. The world of quantum mechanics is one of counterintuitive principles and surprising outcomes, yet it is nearly all proven experimentally, and all of QM is backed up by hours and hours of physicists making calculations. There's even a joke that no undergrad should take a classical mechanics course right before QM. But alas, we will do our best to a) Not break your brain, b) not accidentally kill you of boredom, and c) help you have a basic understanding of QM. First off, the following table shows the conventional Western and Greek letters used for properties and particles.

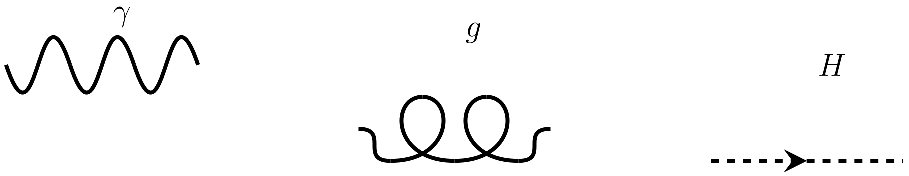
Letter	**Meaning**
c	The speed of light
E	Energy
e^-/e^+	Electron/Positron
g	Gluon
h	Planck's Constant
$\hbar$	Reduced Planck's Constant
i	Imaginary Unit
m	mass
n	Principle Quantum Number[1]
p	Linear momentum
t	Time
v	Velocity
w	Work
V_x	Particle x's Neutrino
α	(alpha) Fine-structure Constant
γ	(gamma) Photon
Δ	(Delta) Change or Uncertainty
δ_{ij}	(delta) Kronecker Delta
λ	(lambda) Wavelength
μ	(mu) Muon
ν	(nu) Frequency
π	(pi) 3.14159...(Irrational)
τ	(tau) Tauon
Ψ	(Psi) The Wave Function

The other important prerequisite for understanding this is the Feynman Diagram. Invented by Richard Feynman, they are special diagrams that represent particle interactions. You can also use these diagrams to make "Feynman Rules" that define how certain particles behave and interact with others. Now let's get into how Feynman Diagrams work. First off, we have a fermion line, with a right-pointing arrow, which indicates a matter particle like a neutrino, or any non-bosonic particle. Fermion lines with left pointing arrows are antimatter.

Next, we'll show two kinds of boson tracks: Normal bosons and gluons.

(a) An electron track. (b) A positron track

Figure 1: 2 Fermionic Particles



(a) A photon/light beam. (b) A gluon track. (c) A Higgs Boson

Figure 2: 3 Bosonic Particles

1 The 17 Particles and the Standard Model

1.1 Matter in the Universe

In the universe, all matter (That interacts with electromagnetic radiation from instruments[2]) is composed of molecules, which contain atoms, which consist of different quantities of electrons, neutrons and protons. Additionally, there exists antimatter, which is composed of the same set of particles, except with opposite charges. Antimatter particles are denoted as their matter equivalents with a bar a across the top. For example, an antimatter muon (μ) would be denoted as $\bar{\mu}$. Electrons are part of the Lepton category of the 17 elementary in what physicists call the "The Standard Model".The reason that neutrons and protons are not in this category is that in 2004, David J. Gross and his colleagues discovered that a proton consisted of two "up" quarks and a "down" quark. This research was later superseded by research that proved that protons did only contain three quarks, but also that there was a threshold at which random strange/anti-strange quark pairs would instantly appear and then annihilate.These quarks bind by the strong nuclear force, which is carried by gluon particles[3].

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[2] This intentionally rules out Dark Matter/Dark Energy, and also Neutrino particles, which we will discuss in the next section.

[3] We will discuss these in the next section.

Quarks	Leptons	Bosons
Up (U)	Electrons (e^-)	Photons (γ)
Down (D)	Electron Neutrinos (v_e)	Gluons (g)
Top (T)	Tauons (τ)	Higgs ($H0, H^+, H^-, h$)
Bottom (B)	Tau-Neutrinos (v_τ)	Z Bosons (Z)
Strange (S)	Muons (μ)	W Bosons ($W-, W+$)
Charm (C)	Muon Neutrinos (v_μ)	Gravitrons[5]

Figure 3: The 17 Particles of the Standard Model

1.2 The 17 Particles and Their Interactions

There are 17 particles known to man that can be divided into 3 categories. These categories are the Quarks, Leptons, and Bosons. Fig. 1 shows the different types of elementary particles. Since neutrons and protons consist of quarks, there are a varying number of electrons orbiting the atom, Quarks and Leptons are the matter particles, and Bosons are particles that carry forces and some, but not all, are massless.[4]

1.2.1 Quarks

Quarks can only exist in small groups bound by a gluon energy exchange[6]. Without this, they lose energy and cease to exist. As per figure 1, there are six different types of quarks. The up, down, top and bottom quarks are all virtually identical and are given those names to differentiate between their interactions and positions within protons, neutrons and other particles. Strange quarks arise from flux tubes, which are total vacuums in the gluon field, where literally nothing exists. The voids in the gluon field, created by two standard quarks[7] exchanging gluons, forms a "bubble," and the outside pressure keeps the particles together, and forms a proton or neutron. We will cover more on Flux Tubes in Section 1.5.

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[4] Massive: Z, W, Higgs
Massless: Gluons, Photons

[6] We will cover gluons in the Bosons and Theoreticals Section

[7] The standard quark group is all quarks but the strange and charm quarks, as well as their antimatter equivalents.

1.2.2 Leptons

Leptons are widely considered to be the "matter," or fermion particles because you can find electrons in every single atom, and muons as well as tauons are very common as well. In the chart of the Standard Model, you will notice that each of the lepton particles has a neutrino equivalent. The term neutron was coined by physicist Enrico Fermi, as a play on words of the Italian word for neutron, "neutrone." Even though neutrinos are part of the "matter" particles, they do not actually interact with EM radiation, and thus are undetectable using traditional instruments. The only ways to detect a neutrino are 1) Detect a weak nuclear force interaction between a neutrino and a gluon, or 2) Detect it as a source of inverse beta decay, given by this equation:

$$\bar{v_e} + p \rightarrow n + e^+$$

It is also important that because electrons, muons and tauons are all leptons, a colliding an electron and positron and can (sometimes) have one of the two following results, shown in Figs. 2.1 and 2.2. The mass of a tauon is greater than that of an electron or muon, so it requires the most energy to produce. The equation below states this.

$$P(\tau) > P(\mu) > P(\gamma)$$

Because:

$$E(\tau) > E(\mu) > E(\gamma)$$

Where $E(x)$ is the energy required to produce particle x.
The reason that a photon beam always connects the two sets of particles is purely because of Conservation of Energy and $E = mc^2$. Conservation of Energy states that in a given closed system, like the aforementioned particle interaction, the net energy is always exactly the same. $E = mc^2$ tells us that mass can be created out of pure energy, which is what light is. So, if the amount of energy in the photon track exceeds the threshold $E(\mu)$, $E(\tau)$ or both, a new set of particles is created. More specifically, we will now look at how different energies produce particles. The arrow represents the creation of particles, and the letters inside the parentheses to the left are the result of the collision.

$$E(e^- + e^+) < E(\mu) \rightarrow \gamma$$

$$\big(E(e^- + e^+) > E(\mu) \text{ and } E(e^- + e^+) < E(\tau)\big) \rightarrow (\mu, \bar{\mu})$$

$$E(e^- + e^+) > E(\tau) \rightarrow (\tau, \bar{\tau})$$

Note: The reason the first situation only produces one particle is because a photon has neutral charge, and therefore is its own antiparticle.

1.2.3 Bosons and Theoreticals

The Boson family of particles is often regarded by the public as well as to some extent, physicists, the "weird" group of particles, or your weird uncle and his booze-loving friends. Most have been proved theoretically and experimentally. For example, the Higgs Boson's theoretical existence was discovered in the 1960s, but wasn't found experimentally until it was discovered in high-energy proton collisions at the LHC. Photons, light particles interact with most other particles, but *not* neutrinos. The Z and W bosons are heavier versions of photons, and they interact. Both of those particles carry the weak nuclear force. The last particle in the boson group is the gluon, which carries the strong nuclear force, and helps bind quarks together, which forms protons and neutrons. Theoretical particles are even more strange. The Tachyon is a particle that can theoretically surpass the speed of light. One way the physicists theorize this is possible is that Tachyons have a property that as their net energy decreases, their velocity increases, which is the exact opposite of traditional bradyonic[8] particles.

1.3 Quantum Fields and Particles

All fundamental particles are parts of a corresponding field, which is everywhere-permeating. If you look at an image of deep space, and removed all matter and antimatter, what you get is nothingness, right? Wrong: Actually, the colloquial definition of nothing, means purely nothing at all. In space, nothing means nothing, except for the quantum field. This situation, however, is entirely hypothetical for a couple of reasons. First, it would be nearly impossible to clear matter (and antimatter) from an area, and second, if you did that, the quantum fields would most likely produce a particle immediately.

1.3.1 The Wave-Particle Relationship

As we mentioned earlier, fields exist everywhere. Particles are simply excitations or kinks in the field. A good analogy here[9], is to think of the quantum field as a

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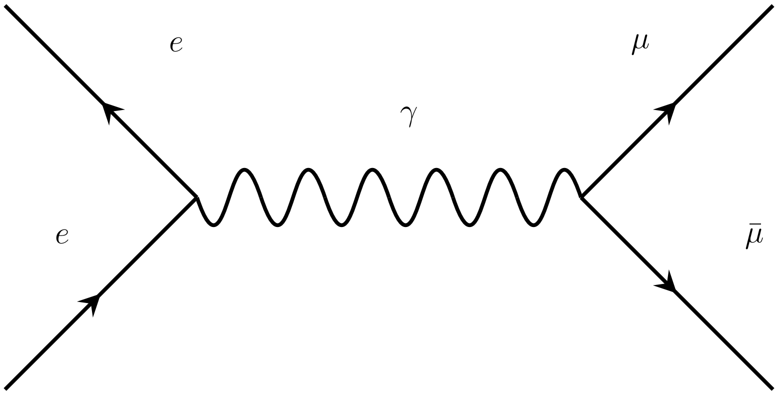
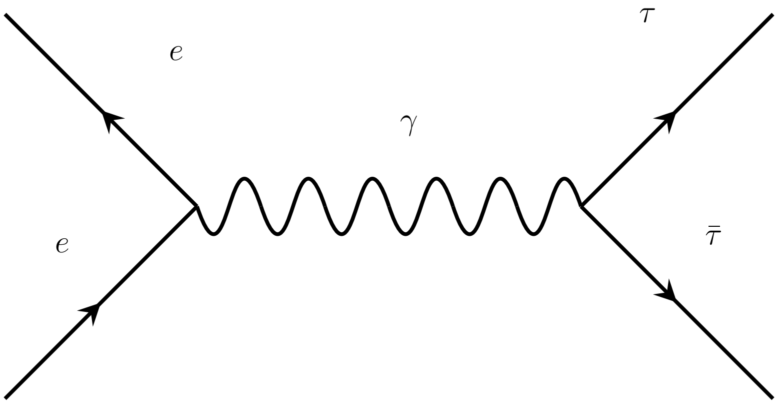
[8] Bradyonic means any particle that travels slower than the speed of light.

[9] See Source 1



(a) Electron-Positron collision producing a muon/anti-muon pair



(b) Electron-Positron collision producing a tauon/anti-tauon pair.

Figure 4: Feynman Diagrams of Lepton-producing electron-positron high-energy collisions

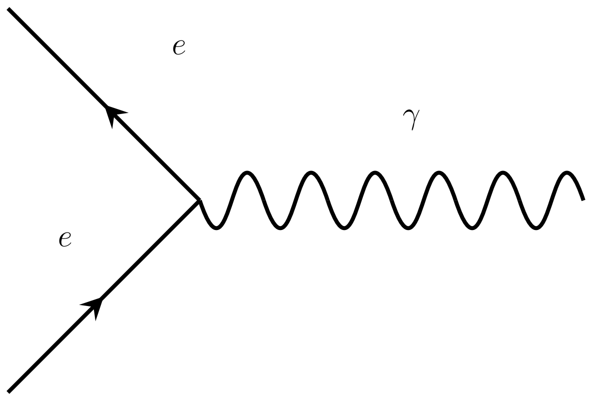


Figure 5: An electron-positron collision producing a photon.

calm ocean. Once energy is added (in this case, a stiff breeze), waves begin to form on the surface. This is quite similar to how particles form out of a quantum field. Each particle in the standard model has its own, unique field.

1.4 The Higgs Boson

Similar to the other particles in the Standard Model, the Higgs Boson is an excitation of the Higgs Field. Contrary to popular belief, the Higgs Boson isnt a new concept. The equations that supported its existence go all the way back to the 1960s when the Standard Model was emerging. However, its true existence wasnt confirmed until 2012, when a team of CERN scientists at the Large Hadron Collider (LHC) found the particle while testing some collisions. The Higgs part of the name comes from the British physicist Peter Higgs (b. 1929). One of its properties is that it gives other particles (especially photons) mass. This happens because photons as well as other particles that enter the Higgs field begin to bounce around (Fig. 2), and decelerate. This interaction is called the Higgs mechanism. As per the proof in Section 3.2, any massless particle must travel at the speed of light, and any massive particle must always travel slower than the aforementioned speed. Thus, as soon as $m > 0, v < c$[10]. In Figure 2, the boson tracks marked by "Z,W" are double tracks of the Z and W Bosons. The outgoing H is the outgoing Higgs particle produced as a result of the interaction. Interestingly, there are actually four different types of Higgs Bosons, and it was only the 4th that caused the scientific uproar in 2012. The

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[10] See $E = mc^2$proof

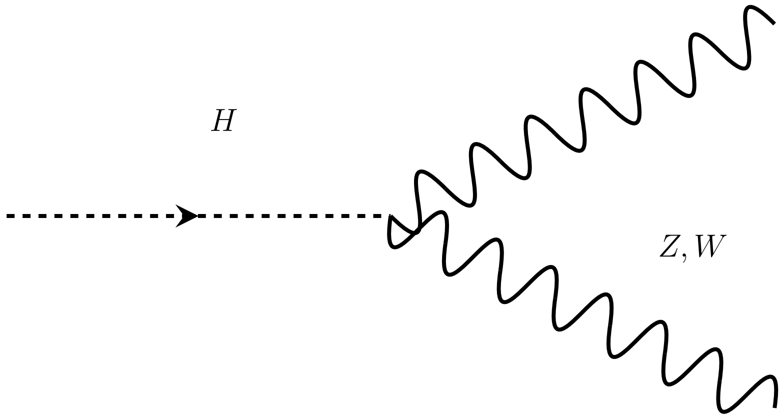

Figure 6: Higgs Boson interaction with Z or W bosons.

four Higgs are represented by $H+, H-, H0$ and h. The first three, due to their polarizations, interact with the W and Z bosons, and get annihilated as per the following Feynman diagram:

1.5 Protons, Neutrons, Gluons and Flux Tubes

In 1991, it was discovered that neutrons and protons are actually composed of quarks. In a neutron, there are two up quarks and a down quark. Protons are the opposite. They contain two down quarks and an up. The different types of quarks are not actually physically different, and their names are used to differentiate. Later in the 1990s, it was determined that protons and neutrons could be represented using Feynman diagrams to show the collisions of gluons, other gluons and quarks. The Proton Feynman diagram states that the quarks are bound by exchanging gluons. As the energy from the collisions surpasses a specific a threshold, strange/anti-strange pairs of quarks appear[11], and then annihilate each other. When the pairing appears, it creates a void, known as a flux tube in the gluon field, and actually creates a vacuum where absolutely nothing exists. This void, according to computer simulations and complex calculations of the Schrödinger equation, the rupture depth of the gluon field does not change,

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[11] This pairing is known as a Meson.

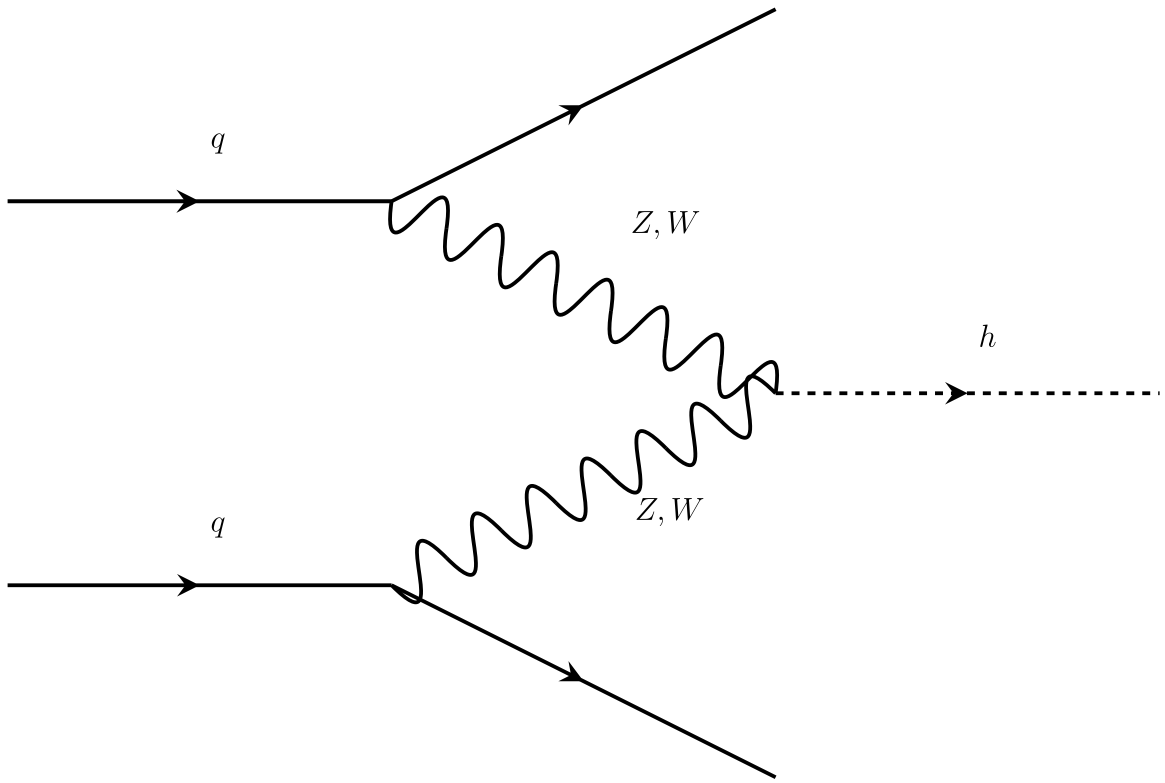


Figure 7: Feynman Diagram for Higgs Boson Interaction

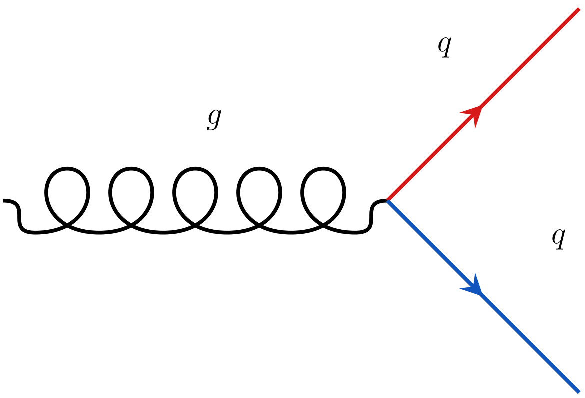


Figure 8: Feynman Diagram of a Gluon binding two quarks.

despite the lengthening of the void on the x-axis. However, this vacuum does not behave like a spring, and there is no elastic force. As the void increases in volume, it generates and energy field, which in turn creates another quark/anti-quark pair, within the flux tube. The annihilation then uses up energy, which brings the proton's total energy below the threshold, and the process resets. Interestingly, the existence of quarks was predicted in the 1960s, but not experimentally proven until the 1990s.In conclusion, the most accurate model of a proton is two up quarks and a down, but also strange/anti-strange pairs popping in and out of existence.This phenomenon creates many flux tubes that combine, and create larger voids in the gluon field. This hole, and the walls that form as a result of its existence confine the quarks to a certain area, which forms the proton. However, this system of quarks would only account for 1% of the proton's mass. However, looking at $E = mc^2$, we can tell that mass can be created as a result of a large quantity of energy. This is given by the equation:

$$m = \frac{E}{c^2}$$

This property, that incredibly high quantities of energy can create mass, compensates for just about 99% of the proton or neutron's mass.

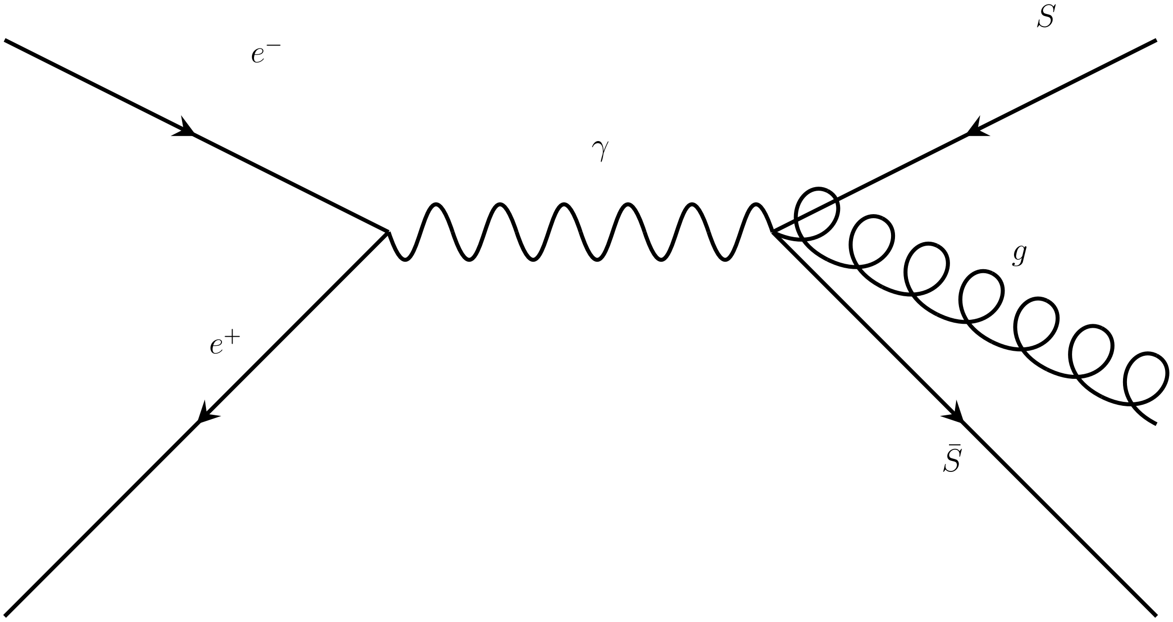
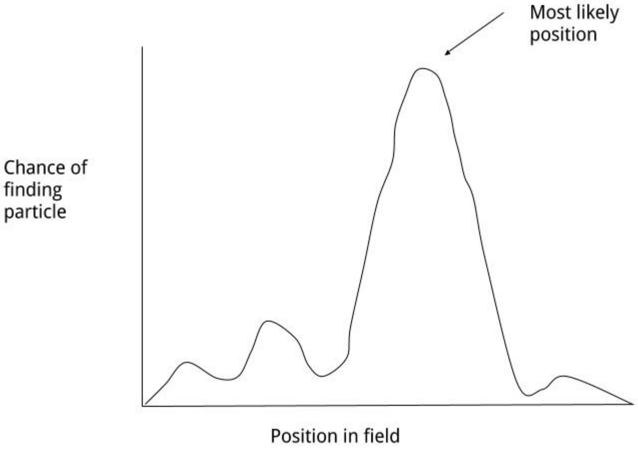


Figure 9: Feynman Diagram of a Flux Tube Annihilation

2 Properties of Particles

2.1 The Wave-Particle Duality

Initially, scientists believed that light was only a wave. That would explain wave interference patterns, which are the complex interactions between waves that collide. However, in 1905, Einstein proposed that light could also be described as discrete chunks of pure energy, which he called quanta, hence the name Quantum Mechanics. This quanta would eventually be renamed photons. Later, French physicist de Broglie expanded this theory with the de Broglie conjecture, which stated that all elementary particles exhibited this duality. His conjecture has been proven experimentally numerous times. Scientists proved this when they did a series of experiments where they shined a light onto complex metals and noticed that the interaction between light and electrons was giving energy to the electrons in little packets, known as quanta. This phenomenon of the transfer of energy is known as the photoelectric effect. Einsteins discovery of the Wave-Particle Duality gave rise to a paper written in 1928, by Danish physicist Niels Bohr, detailing the Complementarity Principle. This principle states that



as a result of the Wave-Particle Duality, an object can behave like both, but when measured, it falls into one or the other, similar to Schrödingers Cat. The most common example of this is a thought experiment in which one launches a singular electron at a wall, and naturally, its waveform radiates outward. On the wall, there is a screen that contains thousands of collision detectors and only one of them goes off. This tells us that the wave has an infinitely growing circumference, but shrinks back into a particle upon measurement. Additionally, if you look at the waveform of a field, the peaks of the waves will tell you where you are most likely to find the particle. See above. The equation for the probability of detecting a particle at a certain x is as follows:

$$P(x) = |\Psi(x)|^2$$

2.1.1 Wavelength Proportionality

All objects have a wavelength (the wave part), but they still exhibit particle behaviors. The wavelength of an object is inversely proportional to its mass. This is why we dont experience wavelength in daily life. However, subatomic and elementary particles have a much larger wavelength because they have a much smaller mass.

2.1.2 The Electron Double Slit Experiment

The Electron Double Slit Experiment (EDSE) was a famous thought experiment first proposed by Richard Feynman. We have two walls, which cannot be penetrated by electrons, one in front of the other. The first wall has two slits in it, that allow electrons to pass through. If we fire electrons in the direction of the slits, electrons that make it through to the rear wall are detectable in a single location with no momentum (so no uncertainty of location). You then repeat this process hundreds of times so that you have hundreds of collisions on the rear wall. This represents a wave in multiple ways. First, if you model two waves originating from the slits, the frequency of electron collisions along the rear wall is directly related to the angle and position at which the two aforementioned waves would collide.

Secondly, the frequency of electrons along the rear wall, when graphed on a histogram, creates wave patterns with its peak at the center, and slowly dissipates, and creates interference patterns that indicate the wave behavior of electrons. The amplitudes of each peak and trough either cancel out or multiply, generating more waves. Looking at the wave pattern, it is vertically symmetrical. Finally, when you block one of the slits, the waves go away. This proves the existence of superposition, or the idea that each electron is passing through both slits at the same time. However, once you track each electron, it is only detectable at one of the two slits. The following image is an electron biprism detector image of the wave pattern of the electrons on the rear surface: As time proceeds from image A through D, the electrons build up and form the interference patterns. These electrons are accelerated to about 120,000 km/s, or about 40 percent the speed of light. This means that the electrons shoot from the emitter to the detector plate in about $\frac{1}{100{,}000}$ of a second. At stage D, the screen had been exposed for 20 minutes. The numbers of electrons in each frame are A - 8, B - 270, C - 2000, and D - 160,000.

2.1.3 The Buckyball Double Slit Experiment

This experiment is similar to the electron double slit experiment but instead of subatomic particles inferring with themselves, macromolecules were used. With this experiment, scientists were able to aim and fire a single buckyball[12] gun. When the buckyballs were fired in the slits, they behaved the same way as the electrons in the electron double slit experiment. This proves that the very odd

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[12] Buckyballs are molecules of 60 carbon atoms (C_{60})

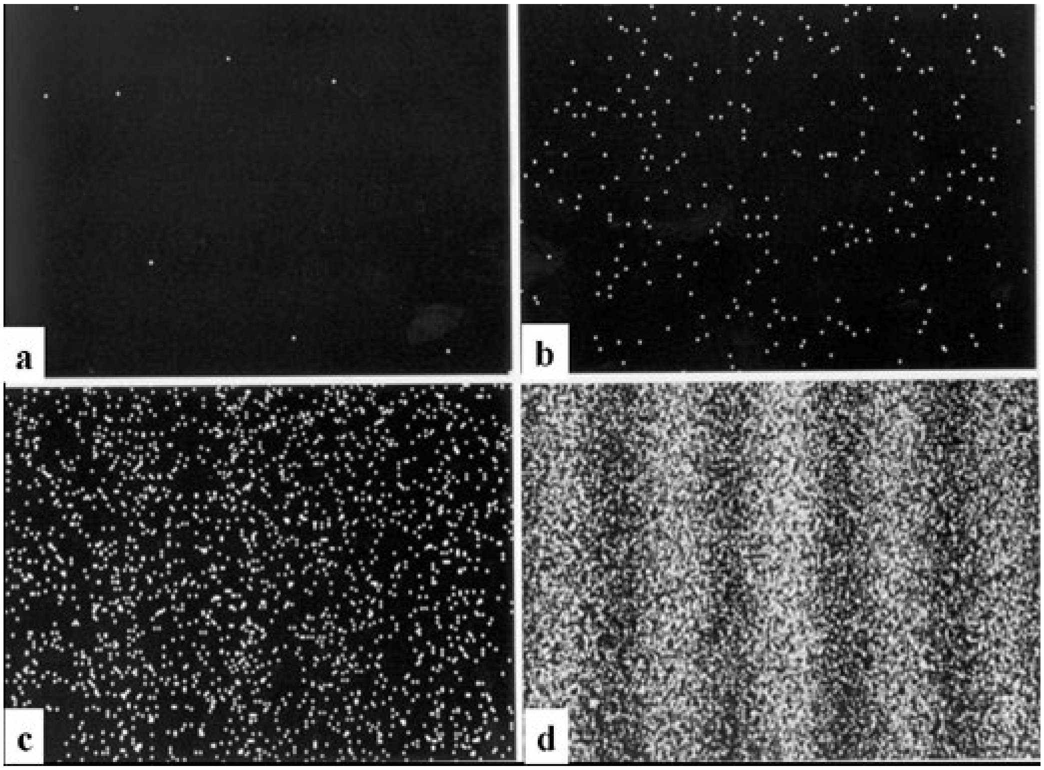

Figure 10: Three images taken from different stages of the experiment.

and strange world of particle mechanics also applies to much larger objects, like molecules. This experiment was conducted by the same team at Hitachi as the above EDSE.

2.2 The Heisenberg Uncertainty Principle

To understand the Uncertainty Principle, it is essential to understand that momentum is inversely related to wavelength. Since p=mv small objects moving very fast, and large objects moving much slower will both produce short wavelengths. For example, a lightly tossed baseballs wavelength is represented by $1 \times 10^{-33} m$. Small enough objects, like electrons and quarks, however, have a wavelength that is more noticeable and measurable. By looking at an objects wavelength, you can determine its momentum, but by measuring the wavelength, you sacrifice some knowledge on position. If you look at the exact position, you have no wavelength, and thus no momentum. You can think of it this way: You can know a cars speed by measuring the time it takes to go a distance x, and then weighing it to get mass, so you can determine momentum. However, by measuring the speed, you become uncertain about location. Additionally, to know location, you have to freeze time so that you can measure it, and all data about speed is lost in that instant. However, despite this baffling paradox, there is one main workaround that allows you to be relatively accurate on both measurements, which is Wave Packeting

2.2.1 Wave Packeting

Wave Packeting is the process of laying over each other waveforms with different lengths, so they are slightly out of phase, and looking at the positions at which the peaks line up. The locations where the peaks and troughs line up increases the amplitude in a certain area, and everything slowly cancels out. In doing this, you give the particle a rough range of different momentum, sacrificing certainty, but getting a positional result that is very close to accurate. But, because of Zenos Paradox[13] and the Principle itself, you can never get the position exactly right. Likewise, the more waves you add, the more certain you become. Doing this creates a wave packet. If you want to be more certain about the position, you need a smaller wave packet, which will cause less momentum certainty. If

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[13] Zeno's Paradox states that no matter how many times you divide a number n by and number x, you will never reach exactly zero, but become infinitely close.

Cyanide?	**Cat Sight?**
Yes	Yes
Yes	No
No	Yes
No	No

Figure 11: Chart of possible outcomes of the Schrödinger's Cat Experiment.

you want more accurate momentum, you need a larger wave packet, which means less positional certainty.

2.3 Schrödinger's Cat: Superposition and Entanglement

Austrian physicist Erwin Schrödinger came up with one of the most famous thought experiments in history. Though there are a few different versions of it; the following is what is believed to be the original: A physicist puts a cat in a box along with a radioactive isotope with a 50 percent chance of decaying. If a Geiger counter that is hooked up to hammer on top of a bottle of cyanide detects radiation from the isotope, a motor swings the hammer, smashing the bottle, and killing the cat. According to Schrdinger, before we look inside, the cat is in what is known as a superpositional state, both alive and dead at the same time. This sounds absurd, and it was for Schrödinger himself. He ended up giving up quantum mechanics to study biology. Absurd as it may seem, Schrödinger was right. This property has been documented in which a certain property of a particle, like which direction it is rotating or moving, is actually superpositional until measured. In further experiments, this effect, were scientists take some property, like which direction the particle is rotating, and have found proof for superposition. Similar to the concept of superposition is Quantum Entanglement. Going back to Schrödinger's Cat, imagine that we look at this from the cats perspective. The cat can either see the cyanide go off and die, or not see it and not die, but no other combinations of the two. (Fig. 3).

Thus, we can cross out the non-corresponding scenarios (Yes-No, No-Yes) because they cant physically happen, and those properties become entangled. Scientists have actually documented a very odd phenomenon in which two particles, no matter how far apart, will always have measurements that correspond. However, in order for this to happen, information would have to travel hundreds of times faster than the speed of light. When Einstein discovered this, he and

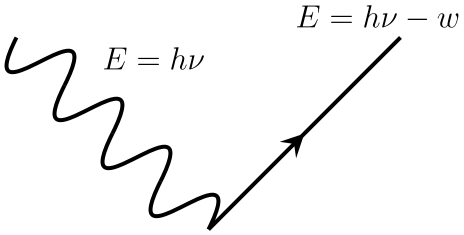

Figure 12: Feynman Diagram of the Photoelectric Effect. The x and y are x and y in this case, not x and t.

his colleagues, Nathan Rosen and Boris Podolsky published a paper on the EPR Paradox (Einstein-Podolsky-Rosen), in which they explained that communication between particles faster than the speed of light was impossible. Einstein described this proposed phenomenon as something in German that translates to something like, "Spooky action at a distance." The paper went on to explain that there must be a yet-undiscovered branch of quantum physics that could explain this phenomenon. At the time, another group of physicists, lead by Niels Bohr, believed that the states were predetermined. Some experiments in the late 20th century made progress on, but did not fully resolve the paradox. These experiments concluded that when two daughter particles formed from one decaying parent, they showed immediate entanglement, which could suggest something about how entanglement actually occurs.

2.4 Planck's Constant and the Photoelectric Effect

In 1900, German physicist Max Planck came up with a number known now as Planck's Constant, which is equal to $6.62607004(81)10^{-34} J \times s$, and is represented by h. $J \times s$ is joule-seconds[14]. The photoelectric effect is a phenomenon in which light hitting a (usually) metal surface, causes the emission of the electrons. As documented by Einstein in a 1905 paper (mentioned above) light comes in packets with definite energies he called quanta[15]. The energy of these is given by:

$$E = h\nu$$

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[14] This is the SI unit of spin in QM.

[15] These were later renamed photons

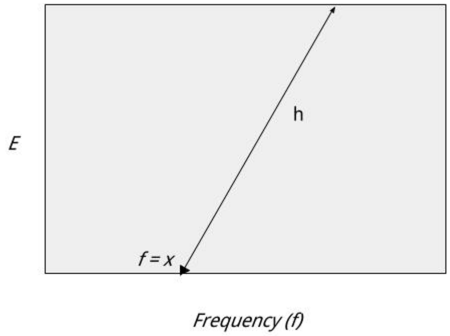

Figure 13: Graph of the energy of excited electrons as a result of light interaction.

then leads to the electrons kinetic energy (after the collision) to be defined as:

$$E = h\nu - w$$

Where $w =$ work required to excite an electron off of its atomic shell[16].

Because of this relationship, below a certain value for $\nu, E = -w$, which is impossible, because kinetic energy is a strictly positive quantity. This means that that below a critical frequency, the photoelectric phenomenon cannot take place. If you look at the above equation, you will notice that it is in $y = mx + b$ (slope-intercept) form. Thus we can graph it like we do in Fig. 11

The equation of this line is $y = h\nu - w$. This means that at a certain point marked $x, y > 0$,and the line continues linearly with a slope h, which is Planck's Constant.

2.4.1 De Broglie Relations

Additionally, since

$$\nu = \frac{c}{\lambda}$$

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[16] You can think of this as the work required to "peel" or "pry" the electron off.

And energy can be defined as:

$$E = \frac{hc}{\lambda}$$

So we can derive

$$p = \frac{h}{\lambda}$$

Which states that the momentum of a wave is equal to Planck's Constant over the wavelength. This discovery lead to the confirmation of the Wave-Particle Duality, because this equation means that a wave can have definite momentum, which is a particle property.

2.4.2 The Reduced Planck's Constant

In 1921, physicist Niels Bohr created the reduced Planck Constant $\hbar$, pronounced h-bar, which he used in many other common equations in quantum mechanics, like the one for linear momentum (see prev. page):

$$p = \hbar k$$

Where k is the radial wavenumber[17]*.*

The equation for Bohr's reduced Planck's Constant is as follows.

$$\hbar = \frac{h}{2\pi}$$

2.4.3 The Rutherford-Bohr Model

In 1911, the Rutherford model was proposed, which stated that as electrons orbited the nucleus, and emitted EM radiation. However, this meant they would rapidly lose energy, and spiral inward toward the nucleus in about 16 picoseconds. In 1913, to account for this, Niels Bohr proposed the Rutherford-Bohr model[18], which contained an infinite number of electron orbit paths (with constant, definite energies), called shells, represented by n=1, 2, 3. The shell n=1was the one closest to the nucleus and had the least energy. He also proposed that the only way for energy to change was if an electron jumped from one shell to another. If an electron jumps to from n=2 to n=3, the energy increases, and EM radiation

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[17] See Appendix A.

[18] This model has been superseded by more accurate ones, but the quantum mechanics and the $\Delta E = h\nu$ formula have proven true in the newer models, as well as experimentally.

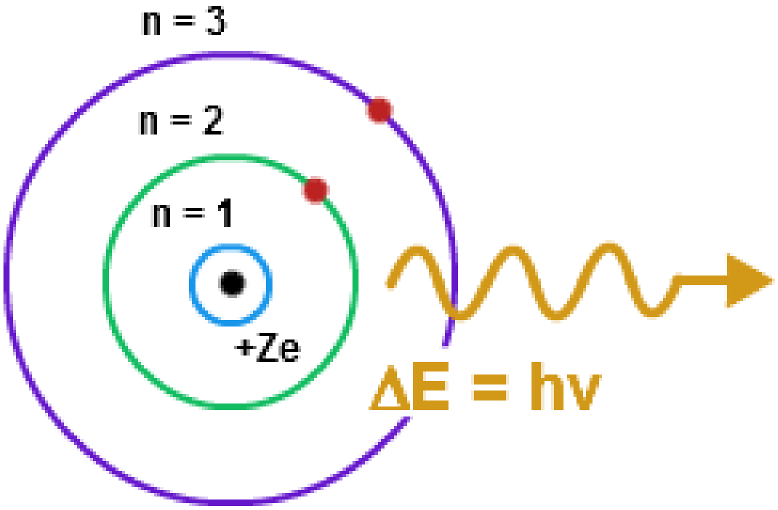

Figure 14: Bohr Model of the Hydrogen Atom

is absorbed. If $\Delta n = n - 1$,the energy decreases, and EM radiation is emitted. Additionally, the angular momentum of an electron in the Bohr atom is always a multiple of the reduced Planck's Constant. This phenomenon adheres to the equation:

$$\Delta E = E_1 - E_2 = h\nu$$

And frequency is defined as it is Newtownian Mechanics:

$$\nu = \frac{1}{t}$$

Where t is the orbit period[19]

2.4.4 Applications of Planck's Constant

Finally, Planck's Constant is also present in the equation that represents Heisenberg's Uncertainty Principle. Given a large set of particles, the calculation of uncertainty of position (x) and of momentum (p) obeys:

$$\Delta x \Delta p \geq \hbar$$

---

[19] The orbit period is the time it takes an electron to orbit its nucleus, which by the $d = rt$ equation is proportional to the electron's energy and and its orbit $(n = 1, 2...)$See above explanation of the Bohr model.

Where uncertainty (Δ) is defined as the standard deviation of the measured to expected value.
The other role that Planck's Constant plays in the Uncertainty Principle is the equation that represents the commutative relationship between the function of position ($\hat{x}$) and the function of momentum ($\hat{p}$).Here, i and j on the x and p take the result of $\hat{x}$ and $\hat{p}$ and plug them into the Kronecker Delta (see footnote).

$$[\hat{x}_i, \hat{p}_j] = -i\hbar\delta_{ij}$$

Where δ_{ij} is the Kronecker Delta[20]

2.5 Non-Persistent States

Using a relatively famous thought experiment, often called the Box Concept, is a good way to understand non-persistent states. First, we assign an electron (or, for that matter any other elementary particle) two quantifiable properties. In this experiment, we will use color and hardness. It is important to note that these are binary properties, as each electron can either be black or white and hard or soft. The other presumption is that the initial set of electrons is 25% black and soft, 25% black and hard, 25% white and soft, and 25% white and hard. Now assume we build two boxes, one that measures colors and the other that measures hardness. These two boxes then dispense the electrons in different directions depending on the outcome of the measurement of the property. The electrons enter at A, and enter the first color box. The black electrons are discarded via B, and the white ones are sent along into the hardness box via C. Note that the electrons moving past C are 100% white. After passing through the hardness box, the hard electrons are filtered out, and the soft ones enter the color box. You would expect them to come out 100% white, since we previously filtered out the black ones, but they come out 50% white to 50% black. This is the principle of non-persistent states. Even though these states are invariable, they can still change. From this experiment, you can conclude that the electrons moving through the apparatus in Fig. 6. are in a superpositional state between each of the measurement outcomes.

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[20] This is a function that takes two inputs (i and j) and returns 1 if they are equal and 0 otherwise. See Appendix B.

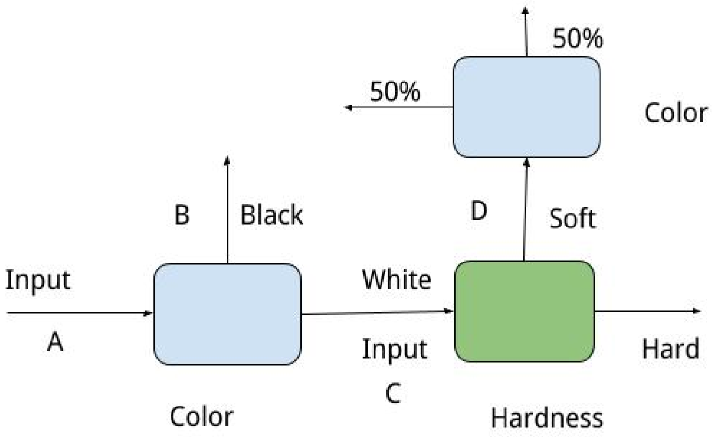


Figure 15: Apparatus for superposition of an electron between 2 outcomes of 2 binary properties

2.6 Bell's Inequality

Bells Inequality is a relatively simple formula derived using logic and classical mechanics, but applies to quantum mechanics. The inequality is as follows:

$$N(A,\bar{B}) + N(B,\bar{C}) \geq N(A,\bar{C})$$

This inequality states that, in a given group of anything (people, electrons, cars etc.) the number of things that have quality A and not quality B plus the number that have quality B and not C are always greater than or equal to the number that have A and not C.

Now, let's convert A, B, and C to three measurable properties of an electron. We will call them X, Y, and Z. The reason for this naming is because X is the electrons angular momentum along in the x-axis, Y is along the y-axis and Z is the z-axis. So now, we plug in the the three new properties into the inequality:

$$N(X,\bar{Y}) + N(Y,\bar{Z}) \geq N(X,\bar{Z})$$

But, as it turns out, this is completely wrong. Many experiments have been conducted, but this inequality does not hold true. So, this tells us that ideas like adding probabilities and Boolean Logic do not apply to the quantum world.

2.7 Spin and Intrinsic Properties

Every particle has the property of spin, which is the angular momentum in the direction of its polarity. For example, electrons and strange quarks both have a spin of $\frac{1}{2}$.Spin has a definite magnitude, as we stated earlier, and a "direction". From this, we can draw a vector to represent spin. But, there is one issue. The "direction" is an angular quantity, not a linear one. Because of this, we can't draw the vector as an arrow, so we draw in a circular shape, and use the radius of said circle to represent the particle's (or vector's) magnitude. Since spin is a strictly positive quantity, the allowed values are given by:

$$S = \hbar\sqrt{s(s+1)} = \frac{h}{4\pi}\sqrt{n(n+1)}$$

Where n is any positive number.

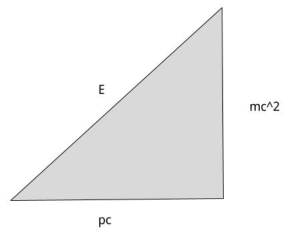


Figure 16: The equation represented as a right triangle and each term of the Pythagorean Theorem substituted for a term in the equation.

3 Einstein and $E = mc^2$

3.1 Where $E = mc^2$ Does not Apply

First off, in Einstein's Theory of Special Relativity, he explained his most famous equation; the relationship between energy of a particle, the particle's mass, and the speed of light (Universal constant). However, you may notice that the equation does not take into account velocity or momentum. Thus, this only applies when the object is stationary, or when p = 0. The below equation shows the full equation if the object has a mass > 0 and momentum > 0.

$$E^2 = (mc^2)^2 + (pc)^2$$

3.2 Proof with the Pythagorean Theorem

Because of the nature of this equation, we can assign the first term a value c, the second a, and the third b. By the Pythagorean Theorem, we can use Euclidean Geometry to represent this equation, and turn it into a right triangle (Fig. 6.) Here, the legs are pc and mc^2 and the hypotenuse is E.[21]

$$a^2 + b^2 = c^2$$

---

[21] Here, c represents the magnitude(length) of the hypotenuse, and not the speed of light.

3.2.1 Scenario 1. Where Particle is Stationary

To prove our initial equation, when the object is stationary, $p = 0$, and the pcterm cancels out leaving us with $E^2 = (mc^2)^2$. By taking the square root, you have the initial equation:

$$\sqrt{E^2} = \sqrt{(mc^2)^2}$$

Which results in, obviously,

$$E = mc^2$$

3.2.2 Scenario 2. Where Particle is Massless

ight, which is the photon particle, is massless. However, when it enters the Higgs Boson field, it bounces around and loses its property of traveling at the speed of light, which gives it mass. We are about to prove why only massless particles can travel at the speed of light. If $m = 0$, then the first term of the equation cancels, leaving us with:

$$E^2 = (pc)^2$$

$$E = pc$$

One may think that a massless particle cannot, by definition, have momentum because momentum is defined as:

$$p = mv$$

However, this formula only applies to much more massive objects. Momentum in quantum mechanics is defined as:

$$p = \frac{\hbar}{i}\Delta = i\hbar\Delta$$

where Δ is the gradient constant and i is the imaginary operator.

3.2.3 Proof Conclusion

The most common way to calculate the velocity of a quantum particle is as follows:

$$v = c \times \frac{pc}{E}$$

Going back to the triangle from earlier, as long as the mc^2side of the triangle exists, the hypotenuse (E) and the other leg (pc) will never be equal, and

$$\frac{pc}{E} \neq 1$$

and thus:

$$v \neq c$$

Because of Einsteinian Special Relativity, we can be more specific by saying:

$$v < c$$

QED

This relationship means that as long as an object has a mass greater than 0, it will never be able to travel at or faster than the speed of light. Additionally, as long as mass equals zero, $v = c$, so light has to forever travel at the speed of light. If it ever decelerates, like passing through the Higgs field, it gains mass, is no longer light, and becomes an electron with negative charge, and more importantly, massive. The interaction between the Higgs Boson and light is where the concept of the Higgs Boson giving mass' to other particles comes from.

A Appendix: Wavenumbers

Also called spatial frequency, it represents one of two things. Either the coefficient of the a wavelength λ per unit distance, like $\bar{v} = 2\lambda/nm$, which represents two cycles with a frequency or radians per unit distance (angular wavenumber) which is represented by k. The equation to calculate wavenumber (in radians) is as follows:

$$k = \frac{2\pi}{\lambda}$$

To calculate the wavenumber in cycles per unit distance:

$$\bar{v} = \frac{1}{\lambda}$$

B Appendix: The Kronecker Delta

See footnote 12 (pg. 15). This function is represented by the Greek letter δ, and is used in many equations (like the commutative relationship equation on pg. 14) in math, engineering, and physics. The symbol previously mentioned is the Greek letter delta, which is used for most delta functions like the Dirac Delta, and is really just a shorthand way to express the function below. Figure

$$\underbrace{\delta_{4,2}}_{0} + \underbrace{\delta_{2,2}}_{1} + \underbrace{\delta_{3,3}}_{1} = 2$$

So:

$$0 + 1 + 1 = 2$$

Figure 17: Example of out-of-context use of the Kronecker Delta.

8 shows a simple use of the Delta, but out of the context of a an equation, like the one on pg. 14. The function equation is as follows:

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

C Appendix: The Speed of an Electron

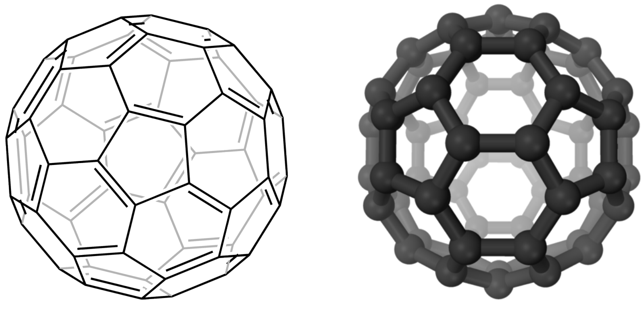
Because of Heisenberg Uncertainty, it is impossible to know the exact speed of an electron orbiting an atom. However we can make very strong estimates using the following model: $v = \alpha c$, where $\alpha = \frac{1}{137.036}$, which is the fine-structure constant. For more info, see the model in the biblography.

D Appendix: Buckyballs

Buckyballs (or buckminsterfullerenes) are fullerene molecules of only carbon atoms (C_{60}) that are arranged into a hexagonal/pentagonal shape, like the pattern on a soccer ball.

E Appendix: Proof of Bell's Inequality

In contrast to what we stated in Section 2.6, here is the classical mechanics proof of Bell's Inequality.



(a) The chemical Structure of a buckyball

(b) A 3-Dimensional rendering of a buckyball.

Figure 18: Two Images of Buckyball Structure

$$N(A,\bar{B}) + N(B,\bar{C}) \geq N(A,\bar{C})$$

The inequality can then be expanded to be:

$$\begin{aligned} \Big(N(A,\bar{B},C) + N(A,B,\bar{C})\Big) &\geq N(A,\bar{B},\bar{C}) \\ +\Big(N(A,\bar{B},\bar{C}) + N(\bar{A},B,C)\Big) &\geq N(A,B,\bar{C}) \end{aligned}$$

By subtracting, we can simplifying, we can cancel out the upper left and lower right, as well as the upper middle and lower left terms, which results in:

$$N(A,B,\bar{C}) + N(\bar{A},B,C) \geq 0$$

QED

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