1. (a) the proof is flawed for many reasons, but the one that jumped out first is that the base case is wrong. $n^2 + 3n$ where $n = 1$ is 4 and therefore even, not odd.
   (b) the proof is correct
   (c)

2. proof by induction on n
   INDUCTION HYPOTHESIS: Assume the claim is true for n=k for some integer $k \geq 2$. In other words, we will assume that the claim is true that for n cities, where every pair of cities is connected by a one way road, there exists a city that can be reached by every other city either directly or via one other city.
   BASE CASE: k=2, there is a one way road from 1 city to the next, thus proving the claim for the base case.
   INDUCTION STATEMENT: we want to prove the claim for $n = k+1$. If we have k+1 cities, and one is taken away, then we are left with k cities which we proved with the base case. Now we take that city and add it back. Now consider adding a city v. In addition to y, we have 3 other types of cities: d which is the current hub which all cities can reach either directly or via 1 city, x which can reach d directly, and a which can reach d by passing through a.
   Thus we can write the current scenario as
   $a \rightarrow x \rightarrow d$
   now, if we introduce v we have multiple cases that are possible.
   Case 1: $v \rightarrow d$
   d remains the hub and the claim holds true.
   Case 2: $d \rightarrow v$ now we need to decide where the road between v and d goes. If $a \rightarrow v$ then v is the new hub and the claim holds true.
   Case 3: $d \rightarrow v \wedge v \rightarrow a$ now we must decide where the road between x and v goes. if $x \rightarrow v$ then v is the new hub.
   Case 4: $d \rightarrow v \wedge v \rightarrow a \wedge v \rightarrow x$. Now we are left with the road between a and d. Now we already assumed that a cannot reach d directly, then that implies that $d \rightarrow a$. So now, a is the new hub. Thus the claim is proven.

3. (a) $\frac{42}{100}$
   (b) $\frac{12}{100}$
   (c) $\frac{88}{100}$
   (d) $\frac{46}{100}$
   (e) $\frac{1}{32}$
   (f) $\frac{1}{32}$
   (g) $\frac{10}{32}$
   (h) $\frac{8}{32}$
   (i) $\frac{31}{32}$

4. There are 3 girls and 1 boy.
   if we draw one girl from the 4, we have a $\frac{3}{4}$ probability of drawing a girl. now we remove that girl, then we have a $\frac{2}{3}$ probability of drawing another girl.
   $\frac{3}{4} * \frac{2}{3} = \frac{1}{2}$