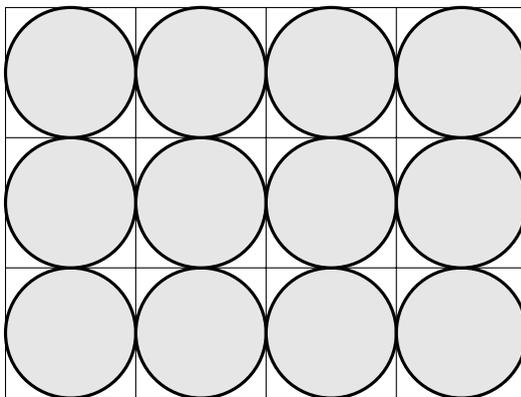


In this project, you will investigate the most economical shape for a can. We're considering food storage cans here. Think soup, tunafish, beans, corn, soda, etc. These are cylinders. For this entire project, use the variable V for the volume of the can, r for the radius of the cylinder, and h for the height. You are free to look up formulas for the volume and area of any shapes that are helpful. You may use a computer program for simplifying algebraic expressions or even to find helpful derivatives, but you should not include information or findings that you do not fully understand, nor skip any steps that you could not justify with computations by hand.

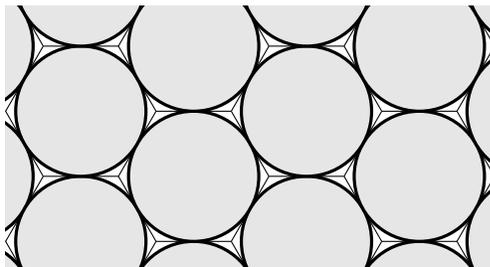
The goal here is to look at a variety of scenarios to try to determine the optimal ratio of the height of a can to its radius, $\frac{h}{r}$. It should work out best to complete these problems in the order they are listed. You do not need to copy the questions word for word in your write-up, nor do you even need to number them. However, a reader should be able to read your report without knowing anything about the project or having access to this document... and you need to answer **all** the questions. This means you need to use your words.

- (1) Measure some cylindrical cans from your cupboard, pantry, or the grocery store. Choose a few different sizes and write down the height and diameter of each. Then, compute the ratio of height to radius. Include pictures of your can examples.
- (2) Suppose that the cost of the metal is the same for the top, bottom, and side of a cylindrical can. Show that the cost for materials is minimized when $h = 2r$. What will cans fitting this description look like when viewed from the side?
- (3) The idea above only considers the cost of the metal that shows up in the final product, but cutting circular tops and bottoms from rectangular sheets means that some of that metal is paid for, but not used. If the tops and bottoms are cut from squares as in the picture below, show that the amount of needed metal is minimized when $\frac{h}{r} = \frac{8}{\pi}$.



- (4) How do cans with these parameters compare to those from Problem 2? Taller? Wider? Skinnier? etc. Why does that make sense?

- (5) A more efficient way of cutting out the disks is obtained by dividing the metal sheets into hexagons rather than squares. If this strategy is chosen, show that the minimal amount of material happens when $\frac{h}{r} = \frac{4\sqrt{3}}{\pi}$.



- (6) How do these cans compares with previous ones? Why does that make sense?
- (7) So far, we've only considered the costs of the materials, not the cost of assembly of those materials. Let's assume that most of the cost in manufacturing is incurred in joining the pieces of metal and that we cut disks from hexagons as in Problem 5. Call the cost per square unit of metal M and the cost per linear unit to join the metal J . Explain why (or show that) the cost to produce one can is given by

$$C = M \left(4\sqrt{3}r^2 + 2\pi rh \right) + J(4\pi r + h).$$

- (8) Show that this cost is minimized when

$$\frac{M\sqrt[3]{V}}{J} = \sqrt[3]{\frac{\pi h}{r}} \cdot \frac{2\pi - h/r}{\pi h/r - 4\sqrt{3}}.$$

- (9) Plot the function $\frac{M\sqrt[3]{V}}{J}$ as a function of $x = \frac{h}{r}$. You will want to replace each $\frac{h}{r}$ on the right side with x , and use that as the independent variable along the horizontal axis of the graph. Which portion of the graph has no meaning in this situation? What are some interesting features of this graph?
- (10) What would a larger value of $\frac{M\sqrt[3]{V}}{J}$ represent in terms of can manufacturing? What does the graph tell us about the optimum value for the ratio $\frac{h}{r}$ in those cases?
- (11) What would a smaller value of $\frac{M\sqrt[3]{V}}{J}$ represent in terms of can manufacturing? What does the graph tell us about the optimum value for the ratio $\frac{h}{r}$ in those cases?
- (12) This analysis should show that larger volume cans should be nearly square (the diameter of the can is very close to its height), while it is more economical to make smaller cans in a tall and thin shape. Make sure that statement agrees with your findings, and compare that to the shapes of the cans in Problem 1. If there are differences, what may have caused that?