Finite Element Method by Example in Qt/C++

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Abstract

Learn FEM by example in a few steps.

1 Introduction

I spent some time lerning FEM using the Internet resources only. I could not find a complete explanation in one place so I decided to create one by putting togather all information I found. Below work is an output of self education. Just curious where I will get from here! All suggestions and comments are welcome!

2 Rotation of Objects

2.1 Rotation of Point 2D

given from geometry:

$$x_0 = rcos\alpha \quad y_0 = rsin\alpha \quad x_1 = rcos(\alpha+\beta) = rcos\alpha cos\beta - rsin\alpha sin\beta \quad y_1 = rsin(\alpha+\beta) = rsin\alpha cos\beta + rcos\alpha sin\beta$$

hence:

$$x_1 = x_0 cos\beta - y_0 sin\beta \quad y_1 = x_0 sin\beta + y_0 cos\beta$$

2.2 Rotation Matrix 3D

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} cos\beta & -sin\beta & 0 \\ sin\beta & cons\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

2.3 Example of Rotated Cube

2.4 Element Stiffness Matrix in Local Coordinates

Relation between axial forces, q1, q2, and axial displacements, u1, u2, only (in local coordinates).

$$\mathbf{k} = \frac{EA}{L} \begin{bmatrix} 1 & -1 - 1 & 1 \end{bmatrix} \mathbf{q} = \mathbf{k} \ \mathbf{u}$$

subsectionCoordinate Transformation
Global and local coordinates

$$L = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2}$$

$$\cos x = \frac{x_b - x_a}{L} = c_x$$

$$\cos y = \frac{y_b - y_a}{L} = c_y$$

$$\cos z = \frac{z_b - z_a}{L} = c_z$$

Displacements

$$u_a = a_x\cos x + a_y\cos y + a_z\cos z \quad u_b = b_x\cos x + b_y\cos y + b_z\cos z \quad \begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} c_x & c_y & c_z & 0 & 0 & 0 \\ 0 & 0 & 0 & c_x & c_y & c_z \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \\ b_x \\ b_y \\ b_z \end{bmatrix} \quad \mathbf{u = T\,v}$$

Forces

$$\begin{bmatrix} f_{ax} \\ f_{ay} \\ f_{az} \\ f_{bx} \\ f_{by} \\ f_{bz} \end{bmatrix} = \begin{bmatrix} c_x & 0 \\ c_y & 0 \\ c_z & 0 \\ 0 & c_x \\ 0 & c_y \\ 0 & c_z \end{bmatrix} \begin{bmatrix} q1 \\ q2 \end{bmatrix} \quad f = T^T q$$

subsectionElement Stiffness Matrix in Global Coordinates

$$\begin{bmatrix} q1 \\ q2 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 -1 & 1 \end{bmatrix} \begin{bmatrix} u1 u2 \end{bmatrix} \quad f = TTqu = Tvq = kuq = kTvTTq = TTkTvf = TTkTvf = KvK = \frac{E}{l}$$

2.5 Stiffness Matrix Construction

Consider linear equations with exactly one solution:

$$a_1 x + b_1 y = c_1 \quad a_2 x + b_2 y = c_2 \quad \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Fixing y to 0 one of constants must be freed (c_2 has been chosen). Moving variables to left and constants to right side we get

$$a_1 x = c_1 \quad a_2 x - c_2 = 0 \quad \begin{bmatrix} a_1 & 0 \\ a_2 & -1 \end{bmatrix} \begin{bmatrix} x \\ c_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ 0 \end{bmatrix}$$
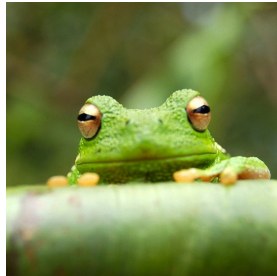
Figure 1: This frog was uploaded to writeLaTeX via the project menu.

Item	Quantity
Widgets	42
Gadgets	13

Table 1: An example table.

2.6 How to Include Figures

First you have to upload the image file (JPEG, PNG or PDF) from your computer to writeLaTeX using the upload link the project menu. Then use the includegraphics command to include it in your document. Use the figure environment and the caption command to add a number and a caption to your figure. See the code for Figure 1 in this section for an example.

2.7 How to Make Tables

Use the table and tabular commands for basic tables — see Table 1, for example.

2.8 How to Write Mathematics

LaTeX is great at typesetting mathematics. Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_i^n X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.

2.9 How to Make Sections and Subsections

Use section and subsection commands to organize your document. LaTeX handles all the formatting and numbering automatically. Use ref and label commands for cross-references.

2.10 How to Make Lists

You can make lists with automatic numbering ...

1. Like this,

2. and like this.

. . . or bullet points . . .

- Like this,

- and like this.

. . . or with words and descriptions . . .

Word Definition

Concept Explanation

Idea Text

We hope you find writeLATEX useful, and please let us know if you have any feedback using the help menu above.